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EDITORIAL

The Experimental Basis of Electromagnetism

AN interesting article with this title, by Norman R. Campbell and L. Hartshorn, was published in the November number of the *Proceedings of the Physical Society*. It is stated to be the first of several sections and it is limited to the consideration of the direct-current circuit. Its object is to develop the working principles of electromagnetism from real experimental facts and not from imaginary experiments such as those with unit poles and point charges, which are "either quite impracticable or incapable of being performed with an accuracy that would be regarded as significant to-day."

The authors differentiate between the familiar definitions, such as that involving the work done in moving unit charges, upon which the mathematical reasoning is based, and the definitions of the magnitudes involved in experimental science, which are capable of measurement to a far higher degree of accuracy. They start with ammeters and voltmeters rather than with "the traditional amber, catskin and lodestone" which are "far less amenable to precise observation." By means of ammeters with blank scales in suitable circuits they verify the addition and subtraction of currents and Kirchhoff's first law, the measurement of resistance by substitution, and the fact that resistance is independent of current, all without any reference to electromotive force or potential difference.

Conductance is also considered before any mention is made of "voltage," a term that the authors appear to prefer to the more expressive "potential difference." Fortunately they stick to the expressive terms "current" and "resistance" and do not introduce "amperage" and "ohmage." By means of voltmeters with blank scales and suitable circuits they then verify the law of addition and subtraction and pass to a consideration of Ohm's law. In the section dealing with electromotive force one is surprised to read that "the well-known Weston standard cell can be regarded as a standard of either e.m.f. or voltage, *though voltage is unquestionably the more fundamental property*." The italics are not in the original but we feel that the statement merits them. We suspect that the word "unquestionably" was introduced to drown any scruples that the authors may have felt, but we may be misunderstanding what is meant by "fundamental." Potential difference is due to distribution of electric charges, and surely the electricity-moving-force, which is the *fons et origo* of this distribution, is more fundamental than the resulting potential difference. The fact that the e.m.f. is determined by measuring the potential difference produced by it does not suggest that it is any less fundamental, but rather more so.

The paper is given a somewhat archaic appearance by the retention of the accent in "ampère" when used as the name of the

unit. This is not only contrary to B.S.I. usage but will not be found in books such as Silvanus Thompson's "Electricity and Magnetism," published last century. Having dropped the final "ay" from Faraday and the final "a" from Volta, one need have no scruples about dropping the accent from Ampère; it is a minor mutilation. In wireman's slang the process is carried a step further and the ampere becomes an "amp".

In subsequent sections it is proposed to deal with varying currents and voltages, inductance and capacitance, the principles of magnetism and electrostatics, and electromagnetic waves. The authors say that they hope to remove from electrostatics and magnetism many of the difficulties that frequently disturb the experimentalist. In conclusion they say, "To the mathematician there are many possible ways of developing the relations of electromagnetism, and they are all exactly equivalent in logic and differ only in elegance. The experimentalist has less latitude; his practice is dictated by stubborn facts that defy representation in equations. The one course he is compelled by circumstances to follow is more important to him than the more elegant courses he might have followed in a better world. This is the excuse we offer to those who find our treatment clumsy".

It is not only the experimentalist who is disturbed by the difficulties in electrostatics and magnetism, and we look forward with interest to the subsequent sections of this paper.

G. W. O. H.

DOPPLER EFFECT IN PROPAGATION*

By H. V. Griffiths

(*Engineer-in-Charge, B.B.C. Receiving Station*)

THE Doppler effect, whereby a change of frequency (and wavelength) of a radiation may occur if the relative positions of source and observer do not remain constant, is a phenomenon which is well known in optics and acoustics. The effect can be used to calculate the velocity of a moving emitter of radiations or of a moving body which is reflecting radiations from a relatively stationary source, the reflected radiation being received by an observer who may be either at rest or in motion with a known velocity.

The Doppler effect has been observed with radio waves which have been reflected from meteors [1] and from rocket projectiles such as the German V2†, on occasions when a relatively "direct" ray was also present. The Doppler effect on the reflected ray caused a detectable heterodyne or difference frequency between the waves arriving by the different paths.

Thus, the above observations—together with optical and astronomical observations and familiar acoustic examples—present examples of the effect in which source and observer are in relative motion. A moving reflector, such as the V2 projectile, presents a case where a twofold effect is produced as, in the primary consideration, the reflector is a moving observer and then "retransmits" the wave as a moving source.

In this respect, it is interesting to note that the equations for moving source and moving observer are not, as might at first be supposed, identical, although they will give approximately identical results if the velocity of relative motion is small compared with the wave-velocity.

A less familiar example of Doppler effect, which is quite commonly noted in long-distance radio transmission, is produced by movement of the reflecting "layer" in the ionosphere, when indirect-ray transmission is involved.

At Tatsfield, in November 1943, the standard frequency transmissions of Washington WWV, which transmits a 15-Mc/s carrier frequency with an error not greater than 1×10^{-8} reference absolute frequency, was commonly received at around 1,600 G.M.T. with an error of $2 - 7$ parts in 10^8, the average being about -3×10^{-8} reference absolute frequency (i.e., -0.5 c/s at 15 Mc/s.)

The following notes discuss the relative

* MS. accepted by the Editor, August, 1946.

† At the B.B.C. Receiving Station at Tatsfield and elsewhere.

mechanics and trigonometry in simple form, as it is believed that published discussions on this subject are not usually accessible easily to radio engineers and that the information needful has not previously been assembled in one paper.

The Doppler Effect

(a) Source in Motion :

A source of radiation, of wave-velocity c and frequency f_s has wavelength λ_s, so that $\lambda_s = \frac{c}{f_s}$ and the observed frequency f_0 when source and observer are relatively stationary will be equal to f_s, assuming a homogeneous medium of propagation.

At the end of the first cycle of radiation, the initial positive peak of the wave will have travelled a distance λ away from the source. But if the source be in motion with velocity v, say towards the observer, then in this period of 1 cycle duration, the source will have moved a distance $\frac{v}{c}\lambda_s$ and the wavelength as measured by the stationary observer will have been decreased by this amount ; i.e., to $\lambda_s(1 - v/c)$.

The number of waves passing the observer each second (i.e., f_0) will have increased, as

$$f_0 = \frac{c}{\lambda_0} = \frac{c}{\lambda_s\left(1 - \frac{v}{c}\right)}$$

$$\text{or } f_0 = \frac{f_s}{1 - \frac{v}{c}} \quad .. \quad .. \quad .. \quad (1)$$

The negative sign in the denominator will apply if the relative motion of the source is toward the observer. If the direction of v be reversed, the sign will be reversed.

(b) Observer in Motion :

Consider the conditions at the head of the previous paragraph when the source and observer were both at rest. Now imagine that the observer begins to move at a velocity v per second toward the source. More waves per second will be received, equal to the number of wavelengths in the distance covered per second ; i.e., an increase of $\frac{v}{\lambda_s}$.

Hence, the observed frequency f_0 will now be $f_s + \frac{v}{\lambda_s} = f_s + f_s\frac{v}{c} = f_s\left(1 + \frac{v}{c}\right)$ .. (2)

The positive sign in the brackets applies to motion towards the source and the sign will be reversed if the direction of motion is reversed.

Now, it will be seen that the two equations for moving source and moving observer, although similar, are not identical. But, if the velocity v is small compared with the wave velocity c, then higher powers of $\frac{v}{c}$ can be neglected. In this case, the two equations will be approximately equal and also f_d, the difference frequency between f_s and f_0, will be approximately equal to

$$f_d = f_s \cdot \frac{v}{c} \quad .. \quad .. \quad .. \quad .. \quad (3)$$

whether the relative motion is from source or observer.

Equations (1) and (2), as shown, differ, although they may be equivalent to a first approximation in the limiting case when v is small compared with c. The reason why the cases differ depending upon whether observer or source is in motion, is at first sight difficult to imagine.

Some assistance in visualizing the difference may be obtained by imagining that, if the source is moving, a wave crest ejected at a particular instant of time has a distance to travel which is " fixed," in that the distance is not changing during the time of transit of the wave crest although the source which ejected it is in motion. On the other hand, in the case of the crest ejected from a stationary source which is to be received by an observer in motion, the distance to be travelled by the wave is constantly changing during its transit. Relative to the wave direction, there is also a difference in the sign of the direction of motion.

In the case of the moving source, the crest emitted at time $t = 0$ will arrive at time $t = \frac{d}{c}$ and a crest emitted at time $t = 1$ (one second later) will arrive at time $t = 1 + \frac{d - v}{c}$. So if the source emits f_s waves per second these f_s waves will be received in $1 + \frac{d - v}{c} - \frac{d}{c}$ seconds $= 1 - \frac{v}{c}$ seconds, and the received frequency will be $f_0 = \frac{f_s}{1 - \frac{v}{c}}$ which is equation (1). But, in the case of the observer being in motion, the wave crest ejected at one instant of time will not

have a fixed distance d to travel. A crest emitted at time $t = 0$ arrives at time $t_1 = \frac{d_1}{c}$ where d_1 represents the distance between source and observer at the time-instant t_1. If the distance at time $t = 0$ was d_0, then $d_1 = d_0 - v\,t_1 = d_0 - vd_1/c$

Thus $d_1 = \dfrac{d_0}{1 + \frac{v}{c}}$ and $t_1 = \dfrac{d_0}{c + v}$

A crest ejected at time $t = 1$ (i.e., one second later) will be received at time $t_2 = 1 + \frac{d_2}{c}$, where d_2 represents the distance at the time-instant t_2; then $d_2 = d_0 - vt_2 =$

$$d_0 - v\left(1 + \frac{d_2}{c}\right) \text{ and } d_2 + \frac{vd_2}{c} = d_0 - v\,;$$

so $d_2 = \dfrac{d_0 - v}{1 + \frac{v}{c}}$ and $t_2 = \dfrac{d_0 - v}{c + v}$

Therefore, the number of crests ejected in one second is received in time

$$t_2 - t_1 = 1 + \frac{d_0 - v}{c + v} - \frac{d_0}{c + v} = 1 - \frac{v}{c + v}$$

and so f_0 (the received frequency)

$$= \frac{f_s}{1 - \frac{v}{c + v}} = f_s\left(1 + \frac{v}{c}\right)$$

which is Equation (2).

Having demonstrated and explained the difference between the two cases of moving source and moving observer, it is interesting to consider what will happen when the velocity of motion in either case becomes almost equal to the wave-velocity c. It will be evident, by substituting $v = c$ in Equations (1) and (2), that when the observer moves toward the source the received frequency will be *twice* the transmitted frequency, whereas with the observer stationary and the source in motion toward him, then the received frequency will tend toward infinity.

Thus, the simplified equation (3) which is a safe approximation to use in either case when v is small compared with c will be increasingly inaccurate as v becomes comparable with c. Fortunately, this approximation can safely be used when the wave under consideration is an electromagnetic or light wave, but with waves of low velocity (e.g., sound) the difference between the two conditions could not be neglected.

(c) Reflection by a moving object

In the case of radiations from a fixed source, which are reflected by an object in relative motion, the reflector can be considered first as a moving observer, and then as a moving source re-transmitting the "first observed" frequency. The Doppler effect will thus change the source frequency in two steps and both equations (1) and (2) will operate together.

Calling the final observed frequency still f_0, then

$$f_0 = f_s \frac{\left(1 \pm \frac{v}{c}\right)}{1 \pm \frac{v}{c}} = f_s\left(1 \pm \frac{v}{c}\right) \cdot \frac{c}{c \mp v} \quad \ldots \quad \ldots \quad (4)$$

So $f_0 = f_s \dfrac{c + v}{c - v}$ if the relative motion is toward the final observer, the signs being reversed if the motion is in the opposite direction.

The difference frequency, $f_d = f_s \dfrac{c + v}{c - v} - f_s$

but, when $v \ll c$, this can be approximated to

$$f_d = \frac{2v}{c} \cdot f_s \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

With this approximation,

$$v = \frac{f_d\,c}{2\,f_s} = \frac{\lambda_s}{2} \cdot f_d \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

and this is the equation for velocity of a moving reflector body, such as a meteor or V2, which is considered to be directly approaching the observer. The same approximation in terms of f_d will apply if the motion be reversed.

Washington Standard Frequency Transmissions

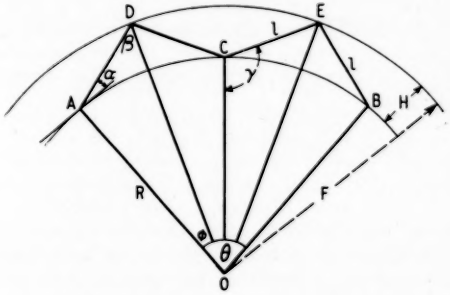
The mean observed f_d of the 15-Mc/s signal at 16.00 G.M.T. in December 1945, was minus 0.5 c/s, by measurement, whereas the transmitted frequency was 15 Mc/s exactly.* A close approximation of what this change of frequency represents in terms of change of path length can be obtained if the geometry of the path taken by the Washington signal can be calculated. Furthermore, it will be possible to relate this measurement to the path in such a way as to determine the rate of change of virtual height of the reflecting layer. Before ob-

* The possible error limits quoted by Washington are $\pm 2 \times 10^{-8}$ but this transmission error has been neglected here.

taining this information from the measured value of f_d, the trigonometry of the reflected ray path between Tatsfield and Washington must first be considered.

Propagation Trigonometry

In the figure, A and B are the points of transmission and reception lying along an arc of the great circle ACB. The angle θ is subtended by the points A and B at the earth's centre, R being the earth's radius. The outer arc joining points D and E is in the ionosphere at the virtual height of reflection, which must be taken as being at a mean distance F from the centre of the earth, although it will not actually be constant either in time or in space, because the ionic density will vary over a long path of this nature, particularly if there is a large difference of longitude between D and E.



The path length L will be the sum of the distances AD, DC, CE, EB, etc., which will vary in number and position with the number of hops (reflections) which the signal undergoes, a two-hop path being illustrated. Thus, there are $2n$ identical triangles involved with a signal taking n hops to reach the observer, each (plane) triangle having an angle at O equal to $\frac{\theta}{2n}$

The radiation and reception angle α is subtended with the tangent line which is at right angles to R. Thus, the angle $DAO = (90 + \alpha)^\circ$ and the angle $ADO = \left(90 - \alpha - \frac{\theta}{2n}\right)^\circ$

If H is the mean height of virtual reflection above the earth's surface, then

$$H = F - R = R\frac{\sin(90-\alpha)^\circ}{\sin\left(90-\alpha-\frac{\theta}{2n}\right)^\circ} - R = R\left[\frac{\sin(90-\alpha)^\circ}{\sin\left(90-\alpha-\frac{\theta}{2n}\right)^\circ} - 1\right] \quad .. \quad .. \quad (7)$$

Similarly, if the total path length $= L$ and the length of one limb of the path $= l$, then the distance $AD = l = \frac{L}{2n}$

Hence $$\frac{L}{2n} = R\frac{\sin\frac{\theta}{2n}}{\sin\left(90-\alpha-\frac{\theta}{2n}\right)^\circ} \quad .. \quad (8)$$

and $$L = 2nR\left[\frac{\sin\frac{\theta}{2n}}{\sin\left(90-\alpha-\frac{\theta}{2n}\right)}\right]$$

Now, in the figure there are six dimensions, of which three must be known in order to solve the others. R and θ are known from geodesy, H can be measured for vertical incidence or for other angles with greater difficulty, by radio-sondage technique. Measurements of the vertical angle of reception (α) can be obtained most easily with a pulsed signal, such measurements having been carried out in India on special B.B.C. pulse transmissions sent out for that purpose in February, 1942. Chamanlal, in India, has also carried out vertical angle measurements of an ordinary carrier signal, using spaced horizontal dipoles, in 1940-42 and 1943[2,3].

The probable degree of error of measurement of both vertical angle and virtual height of reflection is not negligibly small, although the angle measurements with a pulsed signal are fairly accurate and the number of hops taken by the predominant signal can be determined with some certainty in this case. With continuous carrier signals errors may be larger. Virtual height measurements will have some error compared with the true height—even when measured at vertical incidence, because the retardation of the wave in the layer, and the penetration cannot be estimated accurately. We may, however, find it profitable to consider the example of the Doppler effect given earlier, in conjunction with the figure showing propagation path details, which may be combined with hypothetical or calculated values of n, H and α in worked examples, the results from which should be of the right order of magnitude even if they may not be strictly accurate.

To solve the geometry of the propagation path, we shall need to have a measured value of either H or α, as stated above, α being the more convenient because the number of hops taken by the signal will be immediately obvious from the measured value of α. Unfortunately, measurements of α—although possible (e.g., by using spaced-horizontal dipoles connected to the X and Y plates of a cathode-ray tube)—are liable to considerable error. On the other hand, the measured value of virtual height was available for the period in question, so the problem is here presented in the form where H, θ, and of course R, are the known elements.

According to Bennington, the minimum height at 16 00 G.M.T. in November 1945, was approximately 230 km on the western half of the path and 250 km on the eastern half, to which must be added about 30 km for penetration, to obtain virtual height H. In the diagram, H is shown as constant and this simplification will be continued by taking the mean value of $H = 270$ km.

For the solution of this oblique triangle, the equations (7) and (8) already considered are not suitable because the value of α is not known, but other trigonometrical ratios which can be used instead are quite as simply derived.

Putting the angles $\frac{\theta}{2n}$ or $AOD = \phi$, $DAO = \gamma$, and $ADO = \beta$, then there are two methods of solving for γ and β and for l which is the length of the limb between A and D. In the first method, which is the more convenient :

$$\tfrac{1}{2}(\gamma + \beta) = 90^\circ - \tfrac{1}{2}\phi \quad .. \quad .. \quad (9a)$$

$$\tan \tfrac{1}{2}(\gamma - \beta) = \frac{F - R}{F + R} \cdot \tan \tfrac{1}{2}(\gamma + \beta) \quad (9b)$$

Having solved (9a) and, from it, (9b), then

$$l = F + R \cdot \frac{\cos \tfrac{1}{2}(\gamma + \beta)}{\cos \tfrac{1}{2}(\gamma - \beta)} \quad .. \quad (10)$$

follows logically.

In the second method :

$$l = R^2 + F^2 - 2RF \cos \phi \quad .. \quad (11)$$

and from the equation

$$l = R \frac{\sin \phi}{\sin \beta}$$

$$\text{then} \quad \sin \beta = \frac{R \sin \phi}{l} \quad .. \quad .. \quad (12)$$

Putting $H = 270$ km, then $R = 6{,}370$ km and $F = 6{,}640$ km, and $\phi = \frac{35}{2} = 13^\circ 15'$, *for two hops.*

Solving Equations (9)

$$\tfrac{1}{2}(\gamma + \beta) = 90^\circ - 6^\circ 37\tfrac{1}{2}' = 83^\circ 22\tfrac{1}{2}'$$

$$\tan \tfrac{1}{2}(\gamma - \beta) = \frac{6{,}640 - 6{,}370}{6{,}640 + 6{,}370} \tan 83^\circ 22\tfrac{1}{2}'$$

$$= 0.02075 \times 8.54$$

$$= 0.177$$

$$\tfrac{1}{2}(\gamma - \beta) = 10^\circ 02'$$

$$\tfrac{1}{2}(\gamma + \beta) = 83^\circ 22\tfrac{1}{2}'$$

$$\text{then } \gamma = 93^\circ 24\tfrac{1}{2}' \quad \therefore \alpha = 3^\circ 24\tfrac{1}{2}'$$

$$\text{and } \beta = 73^\circ 14\tfrac{1}{2}'$$

And Equation (10)

$$l = 6{,}640 + 6{,}370 \frac{\cos 83^\circ 22\tfrac{1}{2}'}{\cos 10^\circ 02'}$$

$$= 13{,}010 \times 0.1172$$

$$= \text{antilog } 3.183 = 1{,}324 \text{ km}$$

$$\therefore L = 4l = 6{,}096 \text{ km}.$$

So, assuming that the Washington signal had only two reflections, the angle of arrival of the wave α

$$\alpha = \gamma - 90^\circ = 93^\circ 24\tfrac{1}{2}' - 90^\circ = 3^\circ 24\tfrac{1}{2}'$$

If the Washington signal were received by a path having *three reflections*, the corresponding solutions of equations (9) and (10) would become :

$$\gamma = 100^\circ 37\tfrac{1}{2}' \quad \therefore \alpha = 10^\circ 37\tfrac{1}{2}'$$

$$\beta = 70^\circ 32\tfrac{1}{2}'$$

$$\phi = 8^\circ 50'$$

$$l = 1{,}038 \text{ km} \therefore L = 6l = 6{,}228 \text{ km}.$$

Now, reconsidering the Doppler effect on the Washington 1-Mc/s signal, it is obvious that, from Equations (3) or (6), we can calculate the change of path length per second which will produce the observed difference in frequency between the Washington signal as transmitted and as received.

From (3) $f_d = f_s \cdot \frac{v}{c}$, so $v = c\frac{f_d}{f_s} = \lambda f_d$, when λ = wavelength of f_s in metres. .. (15)

In the case of two hops f_d will be one-quarter of the total observed difference frequency because we are considering what happens over the path length l, which is one-half of one hop. Similarly, if the signal were propagated over a three-hop path, then f_d would be one-sixth of the total observed f_d.

Thus the change of path length $v = \lambda f_d =$

$\frac{0.5}{4} \times 20 = 2.5$ metres/second for the 1,524 km (two hops) limb,

$\frac{0.5}{6} \times 20 = 1.6$ metres/second for the 1038 km (three hops) limb.

These two different velocities, 2.5 and 1.6 metres/second represent the change of length of the limb l, in the conditions of propagation considered, this change being caused by the increase in the virtual height of reflection during the time under review. They do not, however, represent the actual rate of change of the virtual reflection-height itself. To obtain this figure from the rate of change of l, the ratio $\frac{\text{rate of change of } H}{\text{rate of change of } l}$ must be calculated, this ratio being itself a variable, dependent upon the variation of the angles γ or β. Rather complex calculations would be required in order to obtain the exact ratio, in the circumstances of several dependent variables being present, the value of f_d and thus of the rate of change of l being, in reality, dependent upon the primary variable, which is the rate of change of H. However, a fair approximation of the ratio can be obtained by inspection; the ratio $\frac{\Delta H}{\Delta l}$ is found to be approximately 2 in the three-hop case and between 2 and 3 in the two-hop case. Applying this ratio, we obtain values for the rate of change of the virtual height of reflection H of *between* 3.2 *and about* 6 *metres per second*, due to the increase in ionic density at about 16.00 G.M.T. in December 1945, calculated from the observed signals on 15 Mc/s. Considerably greater changes of observed frequency have been measured on some occasions: the difference taken in the above example has frequently been exceeded, and the difference-frequency has progressed through zero to the opposite sign in daylight and darkness periods, corresponding with changes in ionic density. For this reason, the assumption made above, that the measured divergence of WWV was not wholly or largely due to differences in frequency standards, is not unreasonable. To confirm this point, simultaneous measurements have been carried out recently, on frequency bands in which propagation paths and conditions were dissimilar. Variations in the percentage divergence so measured were of the expected order of magnitude and of sign.

REFERENCES

[1] "Whistling Meteors—A Doppler Effect." Chamanlal and Venkataraman. *Electrotechnics* (Bangalore) Nov. 1941, pp. 28-39. Also summary in *Nature*, 1942, Vol. 149, p. 416, and further papers *Indian J. Phys.*, Vol. 17, No. 5, p. 239.

[2] Chamanlal. *Indian J. Phys.*, Vol. 15, Part 4, p. 289.

[3] B.B.C. Research Dept. Reports A.010 and A.010/2, July 17, 1942 and May 1, 1944.

NEW BOOKS

Introduction to Electron Optics.

By V. E. Cosslett. Pp. 272+x. Oxford University Press, Amen House, Warwick Square, London, E.C.4. Price 20s.

This book is the outcome of a series of lectures given to students taking an Honours Course in Physics at the University of Oxford.

In the past books on this subject have been either too compendious, or else too limited in size or treatment. Moreover the progress has been so rapid that a book has been out of date almost before it was published. In recent years, however, the subject has shown signs of consolidation, only the practical applications advancing at any great speed. Thus the publication of this book is timely, and due to the concise style of writing, its size is sufficient.

After the introduction there is a chapter on the electrostatic field. Various methods of finding the field shape both theoretically and practically are discussed. Mention is made of the relaxation method of calculation, also of the usual rubber sheet and electrolytic-tank techniques.

There follow chapters on electrostatic and magnetic focusing and one on aberrations of the image. The treatment here is largely trigonometrical, but the logical exposition of the subject is not spoilt by a maze of mathematics, the Hamiltonian method being relegated to the Appendix.

The emission of electrons is covered in 23 pages. Starting with energy states in metals, thermionic-, field- and photo-emission are dealt with, and the chapter concludes with a description of some Image Tubes and Multipliers.

The cathode-ray tube and its derivatives (the Iconoscope, Orthicon, etc.) are grouped together.

A chapter on electron diffraction and the electron microscope has some very fine diagrams and photographs. Under the title "Other Applications: Cylindrical Fields," the broad principles of operation of a number of devices are described, including the Magnetron, Cyclotron, Betatron, and Beam Power Valve. A final chapter covers velocity-modulated electron beams and their applications.

The bibliography is in the form of "Further Reading" at the end of each chapter, and the references given are rather more international than in some previous works on this subject. There is a list of symbols.

Altogether a most excellent book, readable, and useful alike to students and to technicians who require a book of reference.

B. C. F-W.

R.C.A. Technical Papers Index. Vols. I & II (a)

Pp. 143 + vi and 21 + iv. Obtainable free on application to Radio Corporation of America, R.C.A. Laboratories Division, Princeton, N.J., U.S.A.

In this index to papers by R.C.A. authors the arrangement is first, in chronological order; secondly, in alphabetical order; and there are, in addition, author and subject indexes. Vol. I covers the years 1919-1945 and Vol. II (a) deals with 1946.

Principles of Radio for Operators.

By Ralph Atherton, M.S. Pp. 344 + x. Published by Macmillan & Co. Ltd., St. Martin's Street, London, W.C.2. Price 20s.

VERY-WIDE BAND RADIO-FREQUENCY TRANSFORMERS*

By D. Maurice, Ing., E.S.E. (Paris), A.M.I.E.E. and R. H. Minns, A.M.I.E.E.

(British Broadcasting Corporation)

SUMMARY.—The salient features of such transformers are discussed, and as far as possible, quantitative expressions are given which enable the best use to be made of them.

THIS paper attempts to give sufficient information on the design of low-power very-wide band radio-frequency transformers for engineers at present unfamiliar with the necessary technique to acquire a working knowledge of it and to be in a position to initiate transformer designs.

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Introduction

The technique of wide-band r.f. transformer design has advanced considerably in the past few years due largely to the stimulant of war-time requirements. Bandwidths of the order of 200 to 1 are now quite normal requirements (150 kc/s to 30 Mc/s). This width covers the normal range of broadcast frequencies. Recently the upper frequency limit has been raised to cover application to the very high frequencies. A recent design transforms a balanced 100-ohm circuit into an unbalanced 100-ohm circuit at all frequencies between 100 kc/s and 150 Mc/s with a loss of less than 2 db.

The wide variety of applications is indicated by the following few examples :—

800-ohm rhombic aerial to coaxial 100-ohm feeder for a broadcast monitoring service.

100-ohm balanced feeder to 3.33-ohm bus-bars for simultaneous distribution of radio frequencies to a large number of receivers. 100 kc/s to 25 Mc/s. This also forms part of a monitoring service.

Coupling the anode of a KT.44 tetrode to an 80-ohm line and obtaining and feeding 5 watts into the line over the frequency range 200 kc/s to 1,500 kc/s. This is part of the drive equipment of a variable-frequency transmitter.

100 ohms to 3,000 ohms, 40 Mc/s to 60 Mc/s. This was a receiver input transformer used instead of a bandpass-coupled circuit. If carefully designed, the transformer gives greater coupling efficiency than ordinary coupled circuits due to various practical considerations.

100 ohms unbalanced to 500 ohms, unbalanced, covering the range 50 kc/s to 27 Mc/s. This transformer couples a 100-ohm coaxial cable to the aerial and earth terminals of an all-wave receiver. Many such transformers are in service at monitoring centres.

An admittance-measurement bridge-transformer covering the frequency range 100 kc/s to 60 Mc/s.

All these examples deal only with low powers. The application of transformer design principles to high-power equipment remains a development problem, which it is considered, will present no great fundamental difficulties other than the question of insulation breakdowns due to high potential gradients per turn.

2. General

A transformer may be regarded as a device for :—

(*a*) Connecting together impedances of different values.

(*b*) Converting circuits which are unsymmetrical with respect to earth to circuits which are so symmetrical.

(*c*) Isolating circuits which carry direct current from circuits which do not.

(*d*) Obtaining accurate voltage and current ratios.

Most of the above qualities are possessed

MS. accepted by the Editor, October 1946.

by ordinary coupled circuits, but transformers differ from these in that their frequency pass range is much greater than that of the coupled circuits. It is this fact which makes transformer theory so very different from coupled-circuit theory even though transformers are, of course, a special case of coupled circuits. In fact, a transformer might be called a coupled circuit in which the coupling factor is practically unity and having such a wide pass-band that the performance at each end of the range is determined by entirely different physical and electrical constants. As it is this performance which we wish to control we shall sub-divide this paper into two sections. We shall deal first with the factors affecting performance at the lower-frequency limit of the pass range and secondly with the upper-frequency limit.

A simple two-winding transformer, Fig. 1 (*a*), may be represented in greater detail by (*b*), in which R_s and L_s are the resistances and leakage inductances of the windings referred to primary and secondary sides. R_p and L_p are the shunt resistance and shunt inductance referred to the primary side and due to the presence of the core; C_1 and C_2 are the shunt capacitances due to the distributed inter-turn capacitances and any overall capacitance across input and output. side, to be equal to the primary series elements.

In all the applications with which this paper deals the resistance R_s is negligible compared with the reactance of L_s. Fig 1 (*c*) thus simplifies to (*d*). We shall henceforth omit the ideal transformer and consider only one-to-one ratio transformers. In most applications the transformer is designed to work between pure resistances which we shall call R_0.

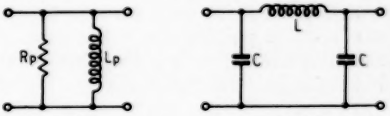

Fig. 2. *At low and high frequencies respectively the transformer equivalent circuit can be reduced to the forms (a) and (b).*

The lower and upper frequency limits of the transformer pass-band will be called f_1 and f_2, respectively.

Now in all wide-band transformers, defined as having an upper cut-off frequency at least five times the lower ($f_2 > 5f_1$), we find $L_s \ll L_p$. Such transformers are so designed that at f_1 the reactance of L_s and the susceptance of C are negligible compared with R_0, whilst the transformer insertion loss is

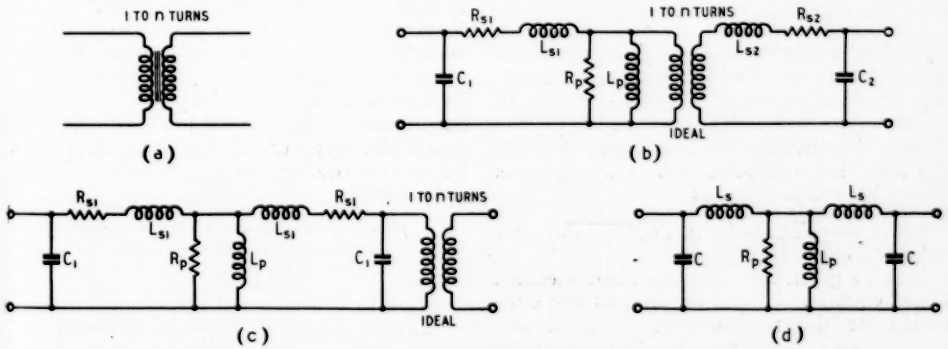

Fig. 1. *A transformer (a) has the equivalent circuit (b) which can be reduced to the form (c). When the effect of R_s is negligible a further simplification to (d) can be made.*

The purpose of the ideal transformer is evident. It might be added that an ideal transformer is one without leakage and shunt admittance. Fig. 1 (*b*) changes to the form (*c*) if we move the ideal transformer to the right, and assume that the windings are sufficiently similar for the secondary series elements, when referred to the primary caused entirely by the admittance of the parallel combination of R_p and L_p. Thus at low frequencies Fig 1 (*d*) becomes Fig. 2 (*a*). In the case of wide-band radio-frequency transformers, particularly those using metallic alloy cores, R_p and the reactance of L_p rise with increasing frequency and become such that their admittance is negligible at f_2.

As the frequency increases, however, the reactance of L_s and the susceptance of C both rise and become predominant at f_2. At f_2, therefore, Fig 1 (*d*) becomes Fig. 2 (*b*) in which $L = 2L_s$.

3. Low-frequency Parameters

3.1. *Shunt Losses.*

The transformer complete with its feed and load, considered at the frequency f_1, is pictured in Fig. 3 (*a*). It has been found in practice that for r.f. work an insertion loss of 2 db is quite permissible. By definition the insertion loss represents the difference in loss between (*a*) and (*b*) of Fig. 3. As the loss of $\frac{e_2}{e_1}$ is 6 db in Fig. 3 (*b*) and we are to admit an insertion loss of 2 db in (*a*), it is clear that the overall voltage gain in (*a*) must be $\frac{e_2}{e_1} = -8$ db. If Q be the ratio of

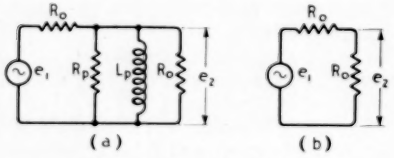

Fig. 3. *The equivalent of the transformer at low frequencies with its generator and load resistances is shown at (a), and the generator and load without the transformer at (b).*

R_p to the reactance of L_p at f_1 it is possible to obtain a relationship between the modulus, $|Z|$, of the open-circuit impedance due to R_p and L_p on the one hand and Q on the other; the entire relation is determined by the fact that the total loss of Fig. 3 (*a*) is 8 db. This expression is

$$|Z| = \frac{2R_0}{9\sqrt{1+Q^2}}\left(4+\sqrt{25+9Q^2}\right) \quad (1)$$

When a given core shape has been decided upon, equation (1) can be used as a specification for the maintenance of good and regular core performance. This may be conveniently done by assuming for test purposes a one-ohm to one-ohm transformer ($R_0 = 1$). The tester should measure R_p and $2\pi f_1 L_p$ and form their ratio, Q. Putting this value of Q into equation (1) gives the required value of $|Z|$. The actual open-circuit impedance per turn squared, as measured, is

$$Z = \frac{R_p}{n^2\sqrt{1+Q^2}} \quad .. \quad .. \quad .. \quad (2)$$

where n is any convenient number of turns wound on the core for test purposes. The number of turns required to give the 2 db insertion loss is:

$$N = \sqrt{\frac{|Z|}{Z}} \quad .. \quad .. \quad .. \quad .. \quad (3)$$

and a suitable upper limit may be placed on this.

This assumes a 1-ohm to 1-ohm transformer. Any impedance level R_0 can be catered for simply by multiplying N by $\sqrt{R_0}$. Equation (1) is plotted in Fig. 4 for ready reference. It should be noted that the method of test allows the greatest possible latitude to the core manufacturer as it does not tie him down to specific values of shunt resistance and reactance, but only to a necessary overall combination of the two. The values of N for different core materials, but constant core shape, plotted against frequency, not only tell the designer how many turns are needed for a given load resistance and a given core material, but also enable him to choose the best core material for the particular value of f_1 that his design requires. Curves of N against frequency for various core materials are shown in Fig. 5. For example, it would be immaterial in a transformer in which the lower-frequency limit was 170 kc/s whether Rhometal or Permalloy C were used, but it would be better to use Mumetal because if, for instance, it was a 100-ohm to 100-ohm transformer then 16 turns would be necessary on a Rhometal or Permalloy C core while only 10 turns would be necessary on the Mumetal core. The lower the number of turns required, the better the core, as this is only used as a turn reducer.

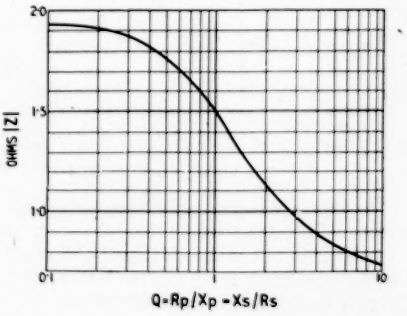

Fig. 4. *Permissible open-circuit impedance of a 1-Ω to 1-Ω transformer for an insertion loss of 2 db.*

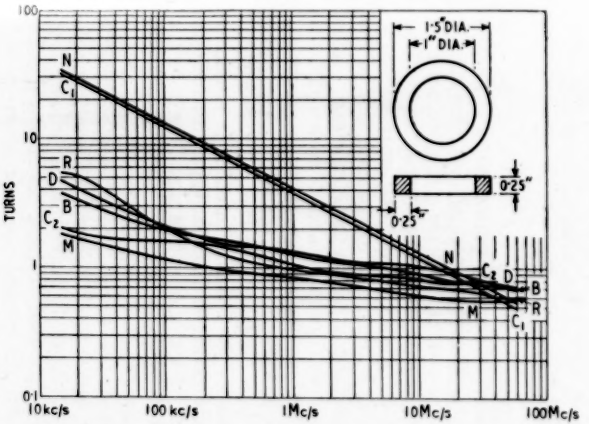

FIG. 5

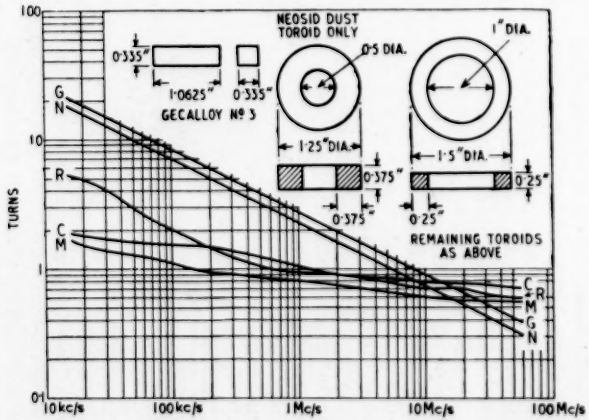

FIG. 6

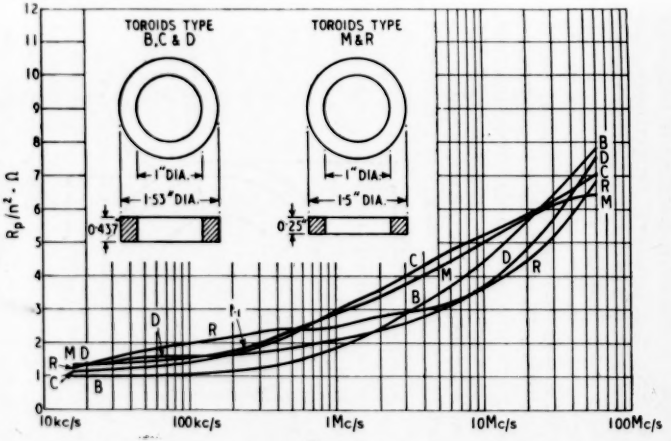

FIG. 7

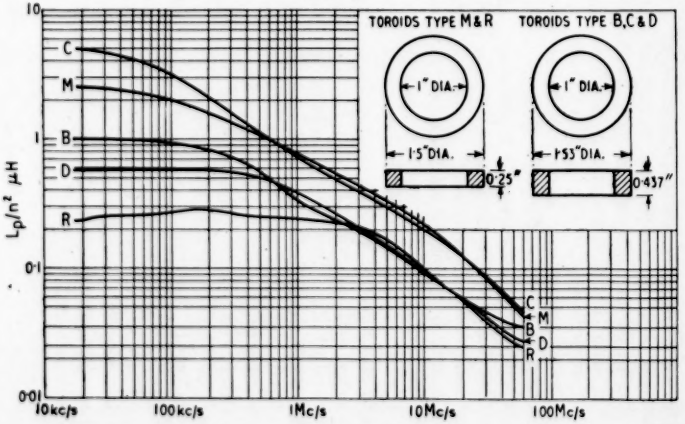

FIG. 8

Figs. 5, 6, 7 and 8. *Comparison of core materials in turns per 1-Ω winding for 2-db loss due to shunt R and X. In the following the figures in brackets refer to the tape thickness in thousandths of an inch. B, Permalloy "B" (2) ; C_1, Dust, Grade C Post Office ; C_2, Permalloy "C" (2) ; D, Permalloy "D" (2) ; M, Mumetal (1.5) ; N, Neosid Dust ; R, Rhometal (2).*

Further, as will be shown later, the leakage inductance is proportional to the square of the number of turns, and an examination of Fig. 2 (*b*) shows that the lower the leakage, L, the higher will be f_2. Fig 5 shows that for v.h.f. transformers dust cores are better than alloy-strip cores. Fig. 6 shows a similar comparison between core materials, but this time the issue is complicated by the fact that only actual marketed cores are shown. It may be observed that the usefulness of dust begins at a lower frequency than in Fig. 5. Little has so far been said about the variation of R_p and L_p with increasing frequency. In practice, though L_p falls with increasing frequency, this increase is less rapid than hyperbolic with the result that R_p and $2\pi fL_p$ actually increase with frequency, the phase angle remaining very approximately constant. Though this increase is slow it is none the less sufficient to reduce the 2-db insertion loss to a quite negligible quantity when $f > 5f_1$ approximately. There is one case in which the cause of insertion loss is of importance. This is when the transformer has to handle higher powers than is usual. A 2-db loss may still be admitted at f_1, but this should be caused in the main by L_p and not R_p as R_p is an indication of the running temperature of the core. The core shape and size dealt with in Fig. 5 will, in the case of curves M and C_2, handle about 1 watt. For powers exceeding this some other material should be chosen such as dust or Rhometal. The same type of core test as described above may be used but higher values of Q should be used than is normal for lower powers. Some curves showing the variations of shunt resistance and inductance against frequency, for various metals and core shapes are shown so that the power handling problem can be tackled more quantitatively. Fig. 7 shows resistance and Fig. 8 inductance. Dust has such a high Q that for our purpose it may be taken as purely reactive.

The measurements made for Figs. 5 to 8 show the characteristics of the cores only. This entailed the use of a rather special type of winding, as most of the cores were toroidal in shape. Consider a winding of bunched turns covering only a small proportion of the total metallic flux path, Fig. 9. The dotted lines indicate leakage flux paths. For a given current in the coil, the ratio of the core flux to the leakage flux will vary as the permeability. This in turn is found to vary more or less inversely as the frequency. If the permeability is high, practically all the flux will travel through the core, and so the inductance L_p of the coil will not vary with the amount of the core covered by the winding. When the permeability is low, as occurs at the higher frequencies, this inductance may well increase with diminishing utilization of the winding space as the inductance of the winding may in large measure be due, not to the core, but to the turns acting as an air-cored solenoid. As is well known, the inductance of such a solenoid becomes greater as the turns are bunched closer together. R_p would increase with turn bunching because it is the core which introduces the losses. In order to eliminate the effects due to different types of windings, therefore, a single turn of metal covering the entire toroid was used. This was split into two discs each having an annular groove the shape of the toroid and of depth half the width of the toroid, Fig. 10. These two discs were arranged to make contact round their outer edges and to have minimum spacing between the central portions when screwed

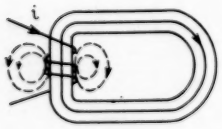

Fig. 9. *Illustrating the leakage flux paths in an iron-cored coil.*

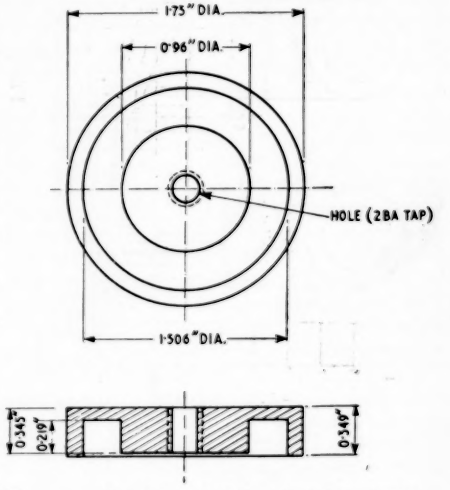

Fig. 10. *Single-turn toroid used in the measurements.*

together by a single central screw which threaded into the farther disc and was insulated from the nearer disc. Connections to this single turn were made by two tag-like washers, one of which made contact with the nearer disc while the other made contact with the under edge of the screw head, the screw being connected with the farther disc and insulated from the nearer. As the discs formed a large mass of high conductivity and unit permeability which closely fitted the toroid, there was little flux outside the core. That flux which contributes to the inductance of the coil, hence to the measurement of the permeability, has a direction tangential to the circular centre line of the core material. Any components of flux which are at right angles to this tangent do not contribute to the measured permeability and are therefore wasted. When the brass discs are present these right-angular components of flux will set up eddy currents which in turn will produce opposing fluxes, the result being an effective increase of permeability.

The above remarks show that the core characteristics are not necessarily fully used unless quite special precautions are taken, precautions which are quite impracticable when it comes to actual designs. It can now be seen that for a given core shape and material there will be a certain value of the frequency below which the permeability will be so high that the form of winding does not matter. Above that frequency, skill and experience are necessary in deciding how much of the core winding space should be

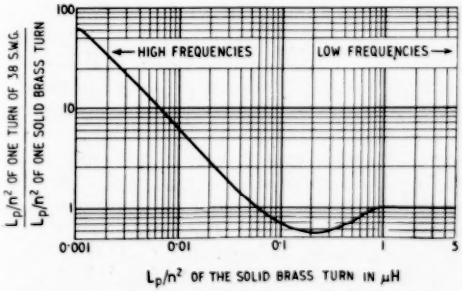

Fig. 11. *Increase in the fraction of the flux not coupling to the core as L_p/n^2 diminishes (with increasing frequency.)*

covered, and if the number of turns is small, what strip width should be used for the windings. Fig. 11 shows the ratio of the open-circuit inductance of a single turn of rather fine wire to that of the single solid turn, as described above, against varying values of the latter obtained by varying the frequency of measurement. The core used for this measurement was the one marked M in Figs. 5, 6, 7 and 8. It may be observed from Fig. 11 that for inductances of the order of 0.2 μH per turn-squared there is a definite advantage in arranging for the winding to cover a large portion of the available winding space. For inductances less than 0.05 μH per turn-squared the core may be regarded electrically as if it were a short

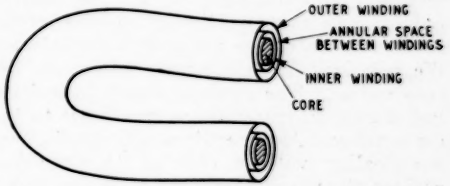

Fig. 12. *Two coaxial single-layer windings are shown.*

stub core. The large values attained by the ratio in Figs. 11 for low values of $\frac{L_p}{n^2}$ of the solid turn, do not indicate that the core is useless, but do show that distributing the winding round it is not always good practice.

All the above discussion may in many cases be quite vitiated by considerations of leakage inductance which will be dealt with later. For example it may, in spite of the foregoing, be desirable to use a spiral winding of flat strip on account of the singularly low-leakage inductances obtained, and to admit of the few extra turns necessary because the winding space is not being fully utilized. It will have been observed that a good deal of the discussion has hinged around cores of toroidal shape. We shall now discuss the question of optimum shape.

3.2. *Core Shape*

Any inductance is inversely proportional to the reluctance offered by the path to the flux passing through it. Inductance is therefore proportional to the cross-sectional area and inversely proportional to the path length. Now consider two single-layer windings, one exactly overlapping the other and entirely filling the winding space of a closed magnetic path, Fig. 12.

A section of core and windings has been removed from Fig. 12 so as to show the general internal configuration. Both the shunt inductance and the leakage inductance

path lengths are the same, namely the total core length, therefore the ratio of shunt to leakage inductance is independent of core length. Now the main linking flux passes through the core cross-section while the leakage flux may be regarded as that part of the total flux which links the inner winding but misses the outer winding. The area through which it passes is therefore that of the annular space between the two windings. The shunt inductance is thus proportional to the square of the core radius (cross-sectional area of core) while the leakage inductance is proportional to the cross-sectional area of the annular space between the windings, therefore proportional to the core radius. The ratio of shunt to leakage inductance is, therefore, proportional to the core radius. Assume for simplicity that we are dealing with a core of Q somewhat greater than unity so that it is the shunt inductance alone which determines f_1. We may write

$$2\pi f_1 L_p \approx R_0, \text{ the load} \quad .. \quad .. \quad (4)$$

Further, it is evident from Fig. 2 (*b*) that

$$2\pi f_2 L \approx R_0 \quad .. \quad .. \quad .. \quad (5)$$

because the leakage reactance is only limited by how much loss can be tolerated at the upper frequency limit. Dividing equation (4) by (5)

$$\frac{f_2}{f_1} \approx \frac{L_p}{L} \quad .. \quad .. \quad .. \quad .. \quad (6)$$

Now replace $\frac{L_p}{L}$ by core radius in (6) and we see that the ratio of the pass-band limits is proportional to the core radius. We therefore want a core with as big a cross-sectional diameter as possible but having any mean length that may be convenient. The above argument applies to cores without derived magnetic paths and wherein the windings cover the whole core length. If the windings cover only a fraction of the total flux-path length then cores with as small a path length as possible should be employed as the path length for the leakage flux will be equal to the winding length, whereas that of the linking flux will be equal to the total core length. The ratio $\frac{f_2}{f_1}$ will only be maximum when the two path lengths are equal.

The most convenient shape for cores fulfilling the above requirements is evidently a toroid of convenient overall diameter and with as large a core diameter as possible. For concentric spiral windings of thin wide copper strip the best core shape is the double-window style usually formed by "E" and "I" laminations or interleaved "E" laminations. The reason that this style is not more popular is the difficulty of obtaining such laminations in very thin (0.002in) material. Considerations relating to shunt capacitance do have an influence on the foregoing, but are not decisive as they usually depend more on the form of the windings than on the actual winding diameter.

Those transformer constants relevant to the performance at f_1 have now been dealt with, and we turn to those which affect performance at f_2.

4. High-frequency Parameters

4.1. *Leakage Inductance.*[1]

It is evident from inspection of Fig. 2 (*b*) that the lower we can make L and C the higher will be f_2, all other things being equal. The number, n, of turns required has been fixed for us by equation (3) and the required working load, R_0. In this section we shall show how to make L as small as possible. Much of the technique which follows is only possible because n is a small number, and this should be constantly

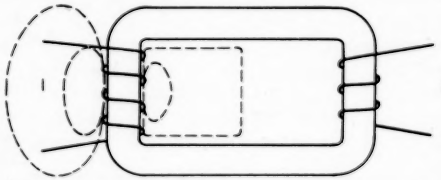

Fig. 13. *Simple two-winding transformer. The dotted lines represent leakage flux.*

borne in mind. It is not proposed to go into the details of calculation of leakage inductance, but by means of figures to illustrate qualitatively, and by algebraic results to give quantitative information useful in transformer design. Consider a simple two-winding transformer, Fig. 13. The dotted lines represent leakage flux. If we neglect the no-load current, as was done in Fig. 2 (*b*), we see that were it not for leakage flux the inductance seen between the terminals 1 would be zero if the terminals 2 were short-circuited. As there is always leakage flux this inductance L is always present. It is calculated by equating $\frac{1}{2}Li^2$ to the total magnetic energy present in the two windings and in the insulation space between them

when one of the windings is short-circuited and the other carries the current i. Consider now one or two practical cases.

Fig. 14 (*a*) shows two concentric helical or spiral windings. If the windings are spiral, then wide strip must be assumed for them. It is assumed that h, the winding width, is large compared with the radial distances between the ends of the windings and the core. In other words the length of the flux paths outside the core is considered to be equal to h only. If n is the number of turns in either winding

$$L = \frac{8\pi^2 n^2}{1{,}000h}\left[\left(\frac{r_1}{3}+\frac{\epsilon_1}{4}\right)\epsilon_1+\left(r_2-\frac{\epsilon}{2}\right)\epsilon+\left(r_2+\frac{\epsilon_2}{4}\right)\frac{\epsilon_2}{3}\right] \quad .. \quad .. \quad .. \quad (7)$$

wherein the units are microhenrys and centimetres.

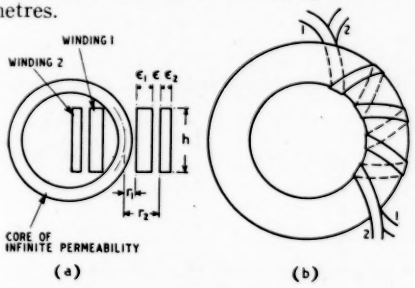

Fig. 14. *Two concentric windings are shown at (a) and a double winding of copper tape at (b).*

If the windings are wound with infinitely thin, wide strip then $\epsilon_1 = \epsilon_2 = 0$, provided that the number of turns on each is small and that the inter-turn insulation is very thin. Equation (7) then becomes

$$L = \frac{4\pi n^2}{1{,}000} \cdot \frac{2\pi\left(r_1+\frac{\epsilon}{2}\right)\epsilon}{h} \quad .. \quad .. \quad (8)$$

The second factor in (8) is simply the cross-section of the annular space between windings divided by its length. Equation (8) will be much used hereafter as it is very simple and sufficiently accurate for many spiral-strip type windings.

Consider now the case wherein the two windings are interleaved turn by turn instead of consisting of two separate blocks as shown in Fig. 14 (*a*). For this case we have

$$L = \frac{8\pi^2 \epsilon}{1{,}000h}\left[n\left(r_1+\frac{\epsilon}{2}\right)+\epsilon\,(n+2)(n-1)\right] \quad .. \quad .. \quad .. \quad (9)$$

wherein the units are in micohenrys and centimetres.

Again the windings are assumed to be wide copper strip of zero thickness separated by inter-turn insulation of thickness ϵ.

If the windings consist of two single concentric layers of helically wound infinitely thin, wide copper strip with spaced turns, Fig. 14 (*b*)

$$L = \frac{8\pi^2 n\epsilon}{1{,}000\,h}\left(r_1+\frac{\epsilon}{2}\right) \quad .. \quad .. \quad (10)$$

wherein the units are in microhenrys and centimetres.

If the region of applicability is carefully adhered to; formulae (8), (9) and (10) give accuracies usually better than 20 per cent, which is adequate. It is now evident that to reduce L to a minimum the winding should be as long as is feasible (h as large as possible), the core and inter-turn insulations should be as thin as possible, as should be the winding strip; and finally the number of turns should be restricted to no more than is essential to satisfy the 2-db shunt losses at f_1.

There is one more way in which L may be reduced and that is by the use of a copper screen between the windings. This applies particularly to toroidal types in which the windings cannot conveniently be made of strip in which corresponding turns fit over one another. For example, in cases where a rather large number of turns is required and

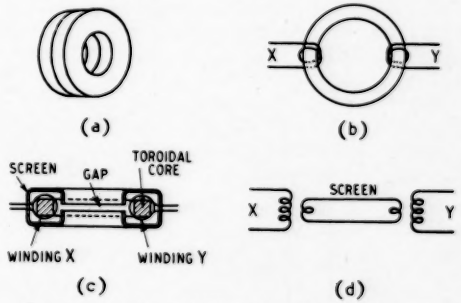

Fig. 15. *The two half-screens of a split screen (a) surround a core with windings X and Y (b) A section is shown at (c) and the equivalent circuit at (d).*

the turns ratio is very different from unity. The action of the split screen may easily be seen from Fig. 15. Here (*a*) shows the two half-screens soldered together round the edges of their outer peripheries. The two windings X and Y are shown at opposite ends of a major diameter, but in fact the

argument also holds if they are spread all round the toroid. The windings may both be inside or outside of the screen or one may be inside. It can immediately be seen from (*d*) that the screen acts as a mutual winding coupled to both X and Y. The screen may also be used to obtain greater coupling of a single winding to the core as indicated by Fig. 10. Screens may also be used to isolate windings from each other but this will be dealt with later.

So far we have dealt with only one of the two major causes of loss in performance at f_2. The other is shunt capacitance, C, Fig. 2 (b).

4.2. *Shunt Capacitance*[2]

This is very dependent upon the type and form of the windings adopted. Formulae will be given which give the overall shunt-capacitance, C, in terms of the component capacitances of which it is made up. Expressions for the individual component capacitances will be given, but these may be subject to large errors for a variety of reasons. Most of the formulae will give results to an accuracy within about 50 per cent, which, though somewhat disappointing, is none the less of great use when exploring paper designs before actually making a try-out. The formulae below were obtained by equating the total dielectric energy $\frac{1}{2}CV^2$ to the sum of the partial dielectric energies.

The case of a single-layer winding on a metal core is shown in Fig. 16 (*a*). The start of the winding is assumed to be at the same potential as the core, that is, the winding is unbalanced with respect to earth. Then

$$C = \frac{n-1}{n^2} C_2 + \frac{(n+1)(2n+1)}{6n} C_1 \quad (11)$$

If $C_1 = C_2$

$$C = \left(\frac{n}{3} + \frac{1}{2} + \frac{7}{6n} - \frac{1}{n^2}\right) C_1 \quad .. \quad (12)$$

If n is large

$$C = \frac{n}{3} C_1 \quad .. \quad .. \quad .. \quad .. \quad (13)$$

With a single-layer balanced winding on a

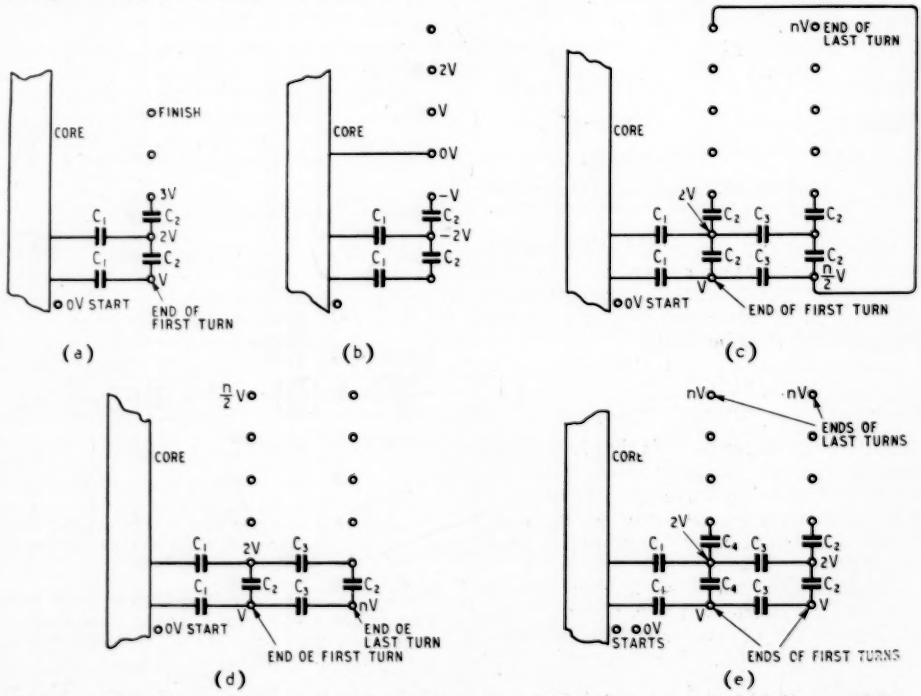

Fig. 16. *Transformer capacitances with a metal core:—(a) single-layer winding; (b) single-layer balanced winding; (c) double-layer winding; (d) double-layer unbalanced winding; (e) two concentric single-layer helical unbalanced windings.*

metal core, Fig. 16 (*b*), the centre of the winding is assumed connected to the core which may be earthed or floating. Then

$$C = \frac{n-2}{n^2} C_2 + \frac{(n+2)(n+1)}{6n} C_1 \quad (14)$$

If $C_1 = C_2$

$$C = \left(\frac{n}{6} + \frac{1}{2} + \frac{7}{6n} - \frac{2}{n^2}\right) C_1 \quad .. \quad (15)$$

If n is large

$$C = \frac{n}{6} C_1 \quad .. \quad .. \quad .. \quad .. \quad (16)$$

A double-layer winding on a metal core is shown in Fig. 16 (*c*). The winding is assumed to be unbalanced; that is, to start at core potential.

$$C = \frac{\left(\frac{n}{2}+1\right)(n+1)}{12n} C_1 + \frac{n-2}{n^2} C_2 + \frac{\left(\frac{n}{2}-1\right)^2}{2n} C_3 \quad .. \quad (17)$$

If $C_1 = C_2 = C_3$

$$C = \left(\frac{n}{6} - \frac{3}{8} + \frac{19}{12n} - \frac{2}{n^2}\right) C_1 \quad .. \quad (18)$$

If n is large

$$C = \frac{n}{6} C_1 \quad .. \quad .. \quad .. \quad .. \quad (19)$$

A double-layer unbalanced winding on a metal core, in which the second-layer is wound back over first layer is shown in Fig. 16 (*d*). For this

$$C = \frac{\left(\frac{n}{2}+1\right)(n+1)}{12n} C_1 + \frac{n-2}{n^2} C_2 + \left[\frac{1}{3n}\left(\frac{n}{2}+1\right)(n-2) + \frac{1}{n^2}\right] C_3 \quad .. \quad (20)$$

If $C_1 = C_2 = C_3$

$$C = \left(\frac{5}{24}n + \frac{1}{8} + \frac{5}{12n} - \frac{1}{n^2}\right) C_1 \quad .. \quad (21)$$

If n is large $C = \frac{5}{24} n C_1$ .. .. (22)

Two concentric single-layer helical unbalanced windings on a metal core, Fig. 16 (*e*) have a capacitance when both windings are in phase of

$$C = \frac{n-1}{n^2}\left(C_2 + C_4\right) + \frac{(n+1)(2n+1)}{6n} C_1 \quad .. \quad .. \quad .. \quad (23)$$

If $C_1 = C_2 = C_3 = C_4$

$$C = \left(\frac{n}{3} + \frac{1}{2} + \frac{13}{6n} - \frac{2}{n^2}\right) C_1 \quad .. \quad (24)$$

In these cases C is the shunt capacitance to earth of either winding.

If n is large $C = \frac{n}{3} C_1$ .. .. (25)

The same construction with windings in anti-phase has a capacitance

$$C = \frac{n-1}{n^2}\left(C_2 + C_4\right) + \frac{(n+1)(2n+1)}{3n}\left(\frac{C_1}{2} + 2C_3\right) \quad (26)$$

If $C_1 = C_2 = C_3 = C_4$

$$C = \left(\frac{5}{3}n + \frac{5}{2} + \frac{17}{6n} - \frac{2}{n^2}\right) C_1 \quad .. \quad (27)$$

If n is large

$$C = \frac{5}{3} n C_1 \quad .. \quad .. \quad .. \quad .. \quad (28)$$

The last winding to be considered is a single spiral of flat strip. This capacitance may be obtained immediately as a special case of a single-layer winding by putting $C_1 = 0$ in equation (11).

$$C = \frac{n-1}{n^2} C_2 \quad .. \quad .. \quad .. \quad (29)$$

It is now necessary to give expressions for the component capacitances C_1, C_2, C_3 and C_4. It should be stated, however, that the principal use of formulae (11) to (29) is in showing the way in which the number of turns and the component capacitances affect the overall shunt capacitance.

To determine C_1, we assume the core to have a circular cross-section for simplicity.[3] Let ϵ_0 be twice the distance between the wire and the core surfaces and assume $\epsilon_0 \ll r$ where r is the core cross-sectional radius in inches. Let d be the wire diameter and K the dielectric constant of the core insulation.

$$C_1 = 0.96 \frac{Kr}{\log_{10} \frac{\sqrt{\epsilon_0^2 + 2d\epsilon_0} + \epsilon_0}{\sqrt{\epsilon_0^2 + 2d\epsilon_0} - \epsilon_0}} \quad .. \quad (30)$$

where C_1 is in picofarads.

For C_2, C_3 and C_4, we let r be the winding radius in inches and assume $r \gg \epsilon'_0$ where ϵ'_0 is the distance between turns (between wire surfaces).

$$C_2 = C_3 = C_4 = 1.9 \frac{Kr}{\log_{10} \frac{\sqrt{\epsilon_0'^2 + 2d\epsilon_0'} + \epsilon_0'}{\sqrt{\epsilon_0'^2 + 2d\epsilon_0'} - \epsilon_0'}} \quad .. \quad .. \quad (31)$$

where C_2, C_3 and C_4 are in picofarads.

(To be continued)

NON-LINEAR REGENERATIVE CIRCUITS*

Frequency and Amplitude Discrimination

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(Post Office Research Station)

1. Introduction

AMPLITUDE discrimination in non-linear circuits is a well-known phenomenon; it must often have been realized that if two tones of unequal level are applied at suitably high levels to a non-linear circuit, then the tone of higher level can considerably reduce the output of the other. However, the effect is not very useful because it is not easy to predict quantitatively with precision, and because it is accompanied by the production of various harmonic and intermodulation frequencies. The application of positive feedback to such a circuit modifies the effect somewhat, and if the feedback is large enough to permit of free oscillation in the absence of an input signal then the amplitude discrimination may be very large at very small levels of input signal and is accompanied by the production of only small amounts of other frequencies. Under these conditions it is a very useful effect. It is necessary to ensure, of course, that no free oscillation does actually occur in the presence of the input signal, and for this reason, the circuit must take the form of a synchronized oscillator. This involves a frequency-discriminating circuit, and the resultant response is therefore a function of this as well as of amplitude discrimination.

In this paper the problem is analysed as that of a synchronized oscillator with an unwanted signal mixed with the injected locking tone. In order to discuss at first only the amplitude discrimination, the unwanted signal is assumed to be of a frequency very close to that of the locking tone; this allows any frequency-dependent effect to be neglected. Later on, the effect of frequency selectivity is briefly discussed.

Before proceeding to the analysis, some previously published discussion[1] of the subject must be referred to. The discrimination of a synchronized oscillator against unwanted signals mixed with the injected synchronizing tone has been shown to be in general a function of two main factors, (1) non-linearity of the amplifier circuit, and (2) the effect of the tuned positive feedback. The previous work considered three frequency ranges, thus:—

(1) Where the unwanted signal has a frequency well removed from the oscillator frequency, so that the effect of the tuned positive feedback can be neglected. The only effect is then non-linearity, and it is shown that the amplification of the unwanted signal is itself linear (because of the small relative amplitude, due to the absence of feedback), but entirely dependent on the amplitude of the forced oscillation at the synchronized frequency. Over ordinary ranges of amplitude, an increase in wanted output causes a reduction of unwanted output.

(2) Where the unwanted signal has a frequency within the range of operation of the tuned positive feedback, but not too close to the oscillator frequency; this last restriction is defined arbitrarily by requiring the unwanted frequency to be outside the range of frequencies over which the oscillator will synchronize. Here, in addition to the non-linear effect mentioned above, which leads to a linear amplification of the unwanted signal, there is a frequency selective response, such that frequencies nearer the synchronized frequency receive some amplification by the tuned positive feedback effect, and are not, therefore, provided with as much discrimination as frequencies further away.

(3) Where the unwanted frequency lies close to the wanted (synchronized) frequency (it is arbitrarily defined as lying within the frequency range over which synchronization occurs), and in consequence both frequencies have comparable amounts of feedback and non-linearity. This case is really the general case, where no real analysis seems practicable. However, by restricting the level of the unwanted signal and assuming it is so close in frequency to the synchronizing frequency that the selective

* MS. accepted by the Editor, July 1946.

effect produced by tuning is negligible, it becomes possible to effect an analysis, and this is, of course, the condition described briefly in the first paragraph. An application which is of importance in radio work for automatic tuning and in other equipment now being developed by the Post Office Research Branch, is the use of a synchronized oscillator to reduce the depth of modulation of a test or carrier tone with a modulated envelope. The modulation may be, say, 30 c/s, or at the most, a few hundred cycles/sec, on a carrier of a few megacycles/sec. For convenience of expression, the analysis which follows is given in terms of the suppression of the modulation in such a case. Some experimental work on this subject has been published by Byard and Eccles,[2] but it does not attempt to explain the results or analyse the problem. The effect is in no way dependent on the unwanted frequencies being sidebands of an amplitude-modulated wave; the result applies for any frequency close to the synchronizing frequency.

2. Analysis

The problem to be analysed may be restated thus:—

A feedback oscillator, as in Fig. 1, has

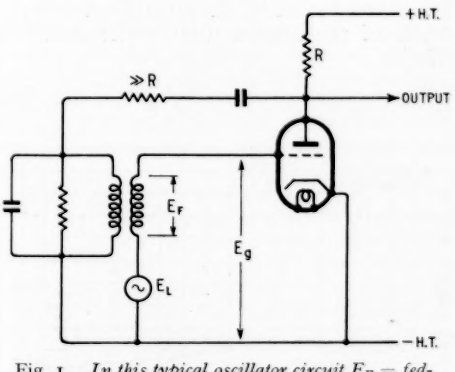

Fig. 1. *In this typical oscillator circuit E_F = feedback voltage, E_L = input, synchronizing or locking voltage and E_g = grid voltage, Suffixes 1 and 2 are used in the text to indicate components of angular frequency ω_1 and ω_2, while the suffix 0 indicates the free oscillation value.*

injected into its grid circuit a tone of frequency very near its natural frequency, modulated to a small depth by a very low frequency such that the sidebands produced by the modulation lie so close in frequency to the tone itself that it can be assumed that the feedback in the oscillator circuit is of substantially constant amplitude response and zero phase for all the frequencies concerned. It is then required to determine by how much the depth of modulation (or the relative amplitude of the sidebands) is reduced in the output by passage through the oscillator circuit.

By small depth of modulation is meant one such that twice the square of the sideband amplitude is small relative to the square of the carrier amplitude (see later). The closeness of the sideband frequencies to the tone frequency is less readily defined, but little error is likely to be produced if the phase angle of the tuned circuit at the sideband frequency does not exceed say, 0.1 radian; this defines the maximum modulating frequency as a proportion of about $\frac{0.05}{Q}$ of the tone frequency, where Q is the magnification factor for the oscillator tuned circuit itself.

2.1 Non-linearity in the Valve as an Amplifier.

Consider two signals $\hat{E}_{g1} \cdot \sin \omega_1 t$ and $\hat{E}_{g2} \cdot \sin \omega_2 t$ applied in series to an amplifier whose instantaneous output voltage e_m is related to its instantaneous input voltage e_i by a power series, thus:—

$$e_m = \sum_{n=o}^{\infty} a_n e_i^n \quad .. \quad .. \quad .. \quad .. \quad (1)$$

For limited ranges of amplitude, a sufficiently close representation is obtained by considering only terms up to the third power. If we do this, and substitute the input signal* $\hat{E}_{g1} \sin \omega_1 t + \hat{E}_{g2} \sin \omega_2 t$ for e_i then we find that the output contains several components, including d.c., sum and difference products, harmonics, and the two input frequencies represented by

$$(a_1 + \tfrac{3}{4} a_3 \hat{E}_{g1}^2 + \tfrac{3}{2} a_3 \hat{E}_{g2}^2) \hat{E}_{g1} \sin \omega_1 t + (a_1 + \tfrac{3}{4} a_3 \hat{E}_{g2}^2 + \tfrac{3}{2} a_3 \hat{E}_{g1}^2) \hat{E}_{g2} \sin \omega_2 t \quad .. \quad .. \quad (2)$$

It will be appreciated that a_3 is negative, so that the amplification for these components is reduced as the amplitudes increase.

If we now make the stipulation that $2\hat{E}_{g2}^2 \ll \hat{E}_{g1}^2$

*If this signal is applied to a grid circuit with a bias voltage E_b, then instead of a_1 in equation (2) we should write $A_1 = a_1 + 2a_2 E_b + 3a_3 E_b^2$. As this term is independent of the signal voltages, the presence of bias in no way affects the working and conclusions of this paper, except that wherever a_1 is written, it should be replaced by A_1.

we see that the amplification to ω_1 is $a_1 + \frac{3}{4} a_3 \hat{E}_{g1}^2$ and the amplification to ω_2 is $a_1 + \frac{3}{2} a_3 \hat{E}_{g1}^2$ .. (3)

The amplification to ω_2 is thus linear ; i.e., independent of its own amplitude.

We can see that this working represents

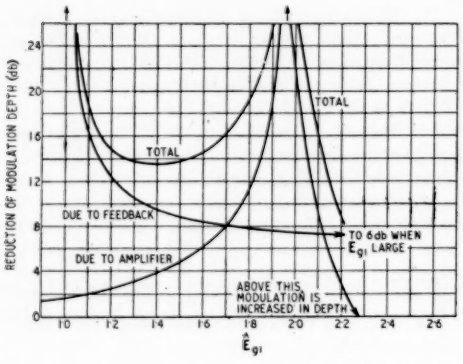

Fig. 2. *The reduction of the modulation depth of the injected signal as a function of grid voltage* ($a_1/a_3 = -5.75$, $\hat{E}_{g0} = 1.0$).

the case of the modulated tone, except that we have considered only one sideband. The sidebands evidently have negligible influence on each other, so the reduction in depth of modulation caused by the passage through the amplifier is by the ratio

$$R_A = \frac{a_1 + \frac{3}{4} a_3 \hat{E}_{g1}^2}{a_1 + \frac{3}{2} a_3 \hat{E}_{g1}^2} \quad .. \quad .. \quad (4)$$

where $\hat{E}_{g1}$ is the peak amplitude of the tone when unmodulated. The absolute values of a_1 and a_3 are of no significance ; it is their ratio which determines R_A.

To see the effect of this it is convenient to take numerical values. For a small r.f. pentode a typical value of a_1/a_3 is -5.75. For this value the ratio R_A is plotted (in decibels) against the peak tone voltage in Fig. 2. It will be seen that the reduction in depth of modulation increases as the voltage increases, up to an infinite point where the amplification to the sideband is zero ; above this the reduction in depth decreases, until after a certain point the depth is actually increased. We must not, however, overlook the fact that this considers only those components of the output which are at the input frequencies ; other frequencies are produced by the non-linear action, and are discussed in Section 2.4.

2.2 *Positive Feedback*

The circuit of the oscillator is shown in Fig. 1, and it is clear that in the equilibrium condition for any frequency the grid voltage E_g is the sum of the input voltage E_L and the fed-back voltage E_F ; i.e.,

$$E_g = E_L + E_F \quad .. \quad .. \quad .. \quad (5)$$

Now for the tone at frequency ω_1 (considering the fundamental component only)

$$\hat{E}_{F1} = (a_1 + \tfrac{3}{4} a_3 \hat{E}^2_{g1}) \hat{E}_{g1}$$

$$\text{i.e., } \hat{E}_{L1} = \hat{E}_{g1} - \hat{E}_{F1}$$

$$= \hat{E}_{g1} (1 - a_1 - \tfrac{3}{4} a_3 \hat{E}_{g1}^2) \quad .. \quad (6)$$

where a_1 and a_3 can now be specifically defined as the loop-gain coefficients. Similarly for the sideband at frequency ω_2

$$\hat{E}_{F2} = (a_1 + \tfrac{3}{2} a_3 \hat{E}_{g1}^2) \hat{E}_{g2}$$

$$\text{i.e., } \hat{E}_{L2} = \hat{E}_{g2} - \hat{E}_{F2}$$

$$= \hat{E}_{g2} (1 - a_1 - \tfrac{3}{2} a_3 \hat{E}_{g1}^2) \quad .. \quad (7)$$

The input modulation ratio is $\dfrac{2\hat{E}_{L2}}{\hat{E}_{L1}} = m_L$

$\therefore$ From (6) and (7),

$$\frac{2\hat{E}_{g2} (1 - a_1 - \frac{3}{2} a_3 \hat{E}_{g1}^2)}{\hat{E}_{g1} (1 - a_1 - \frac{3}{4} a_3 \hat{E}_{g1}^2)} = m_L \quad .. \quad (8)$$

The modulation ratio on the grid, as a result of the feedback, is

$$\frac{2\hat{E}_{g2}}{\hat{E}_{g1}} = m_g \quad .. \quad .. \quad .. \quad .. \quad (9)$$

Thus from (8) and (9), the ratio of reduction in depth of modulation due to the feedback effect is

$$R_F = \frac{m_L}{m_g} = \frac{1 - a_1 - \frac{3}{2} a_3 \hat{E}_{g1}^2}{1 - a_1 - \frac{3}{4} a_3 \hat{E}_{g1}^2} \quad .. \quad (10)$$

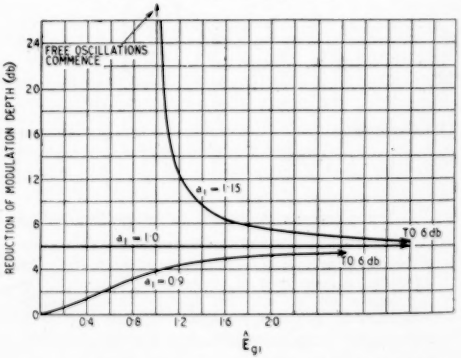

Fig. 3. *The reduction of modulation depth due only to feedback for different amounts of feedback* ($a_1/a_3 = -5.75$).

It is clear that in this case the actual magnitudes of a_1 and a_3 matter, and not only their ratio. A numerical case is shown

in Fig. 2 for values of a_1 and a_3 of 1.15 and -0.2 respectively. These values give the same ratio of a_1/a_3 as was used for illustrating the previous effect, and give a free oscillation amplitude (i.e., the amplitude of the self-oscillation when there is no input) at the grid of $\hat{E}_{g0} = 1.0$. It will be seen that the reduction in depth of modulation is very large when the grid amplitude only slightly exceeds the free oscillation value (i.e., when the input voltage is small), but decreases to an ultimate value of 6 db as the grid voltage (and also the input voltage) is increased.

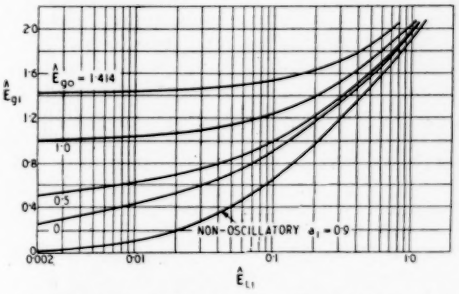

Fig. 4. *The relation between the input, forced, and free oscillation amplitudes* ($a_1/a_3 = -5.75$).

When the output is taken from the anode circuit of the oscillator, the two effects described above are additive (in decibels), as shown in Fig. 2. The minimum reduction in depth of modulation in this example is about 14 db except at very high amplitudes which would not normally be met with in practice on this type of valve.

In Fig. 3 the feedback effect is illustrated for the case of the circuit being *just* capable of self-oscillation ($a_1 = 1.0$), and of being incapable of self-oscillation ($a_1 = 0.9$). The former case gives a constant reduction of 6 db, while the latter gives a reduction rising from zero to 6 db.

2.3 *Relations between Input and Grid Voltages*

So far, for ease in analysis, the effects have been expressed in terms of grid voltage. It is generally more convenient for practical applications to use relationships in terms of input voltage. It is simple enough to relate input and grid voltages, and also to determine the amplitude $\hat{E}_{g0}$ of free or self-oscillation.

We obtain the value of $\hat{E}_{g0}$ as the particular value of $\hat{E}_{g1}$ in Equ. (6) when the input $\hat{E}_{L1} = 0$;

$$\text{i.e., } \hat{E}_{g0} = \sqrt{\frac{1 - a_1}{\frac{3}{4}a_3}} \qquad \ldots \quad \ldots \quad (11)$$

Therefore, for any specified value of $\hat{E}_{g0}$ and given value of a_1/a_3, we can determine the corresponding values of a_1 and a_3. Then by Equ. (6) we can relate the input and grid voltages. This has been shown graphically in Fig. 4, where a family of curves relating $\hat{E}_{L1}$ and $\hat{E}_{g1}$ is given for a number of values of $\hat{E}_{g0}$. A curve is also given for a case where the loop gain is insufficient to maintain self-oscillation (i.e., $a_1 < 1.0$). It will at once be observed that when $\hat{E}_{g0}$ is large, and consequently the valve is being worked very non-linearly, the input voltage, if small, has practically no influence on the grid voltage. This leads to a simple physical explanation of the reduction in depth of modulation; variations in the envelope amplitude of the input signal, produced by the modulation, cause little corresponding variation in the output envelope amplitude.

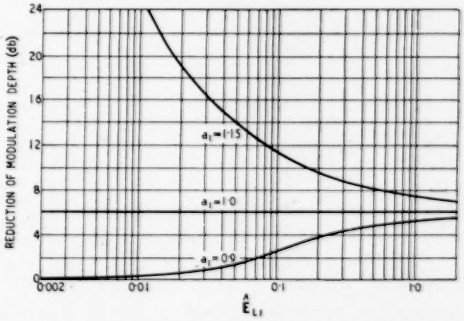

Fig. 5. *The reduction of modulation depth due to feedback expressed as a function of the injected voltage* ($a_1/a_3 = -5.75$).

In Fig. 5 the reduction in depth of modulation due to the feedback effect alone is plotted against input voltage for the three conditions, circuit capable, just capable, and not capable of self-oscillation.* In Fig. 6, the overall reduction (i.e., the sum of the amplifier non-linearity and feedback effects) is similarly plotted against input voltage. It will be seen how satisfactory the synchronized

* It should be noted that if the feedback were negative, or "degenerative," the effect would be opposite for the smaller values of input voltage (i.e., the difference in level between the two tones would be reduced), although to a relatively small extent. At larger input voltages, the effect is similar whether the feedback is positive or negative, and approaches a final value of discrimination of 6 db.

oscillator can be in producing a relatively pure tone from a modulated one, which can be of very low level, or from one mixed with some other unwanted frequency, providing the amplitude of free oscillation is high enough to indicate a reasonable margin of loop gain.

2.4 *Other Frequency Components*

In the foregoing sections only the components at the fundamental frequencies have been considered. A number of other components is produced by the non-linear action, comprising harmonics of the input frequencies, various sum and difference products, and particularly products such as $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. Of all these, only those of the last type fall anywhere near the oscillator frequency, and it is reasonable to assume the tuned circuit of the oscillator reduces the amplitudes of all the others to negligible proportions. Assuming the cubic law, the only frequencies falling near the oscillator frequency are $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. The amplitudes are $-\frac{3}{4} a_3 \hat{E}_{g1}^2 \hat{E}_{g2}$ and $-\frac{3}{4} a_3 \hat{E}_{g1} \hat{E}_{g2}^2$ respectively.

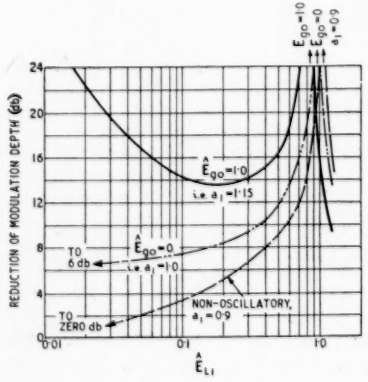

Fig. 6. *Overall reduction of modulation depth for various injected voltages* ($a_1/a_3 = -5.75$).

Since we know $\hat{E}_{g2}^2 \ll \hat{E}_{g1}^2$, the second is obviously very small, leaving the component $-\frac{3}{4} a_3 \hat{E}_{g1}^2 \hat{E}_{g2} \sin(2\omega_1 - \omega_2)t$ as the only one of any significance.

It is evident, then, that the ratio of the amplitude of this component to that of the sideband frequency ω_2 is $-\frac{3}{4} a_3 \hat{E}_{g1}^2$. For a large proportion of practical cases, this ratio will be considerably less than unity, and the third-order component will be of little importance. For large values of $\hat{E}_{g1}$, the amplitude of the new frequency will exceed that of ω_2, and this may be serious; but it must be observed that such cases correspond to large injected voltages. In practice, such large injected voltages are unlikely to be used, as they do not take advantage of the feedback effect and serve no useful purpose.

2.5 *Discrimination over the Whole Frequency Range*

Having shown how the synchronized oscillator provides discrimination on a basis of amplitude non-linearity to an unwanted signal, even when this is of frequency very close to the synchronized frequency, it will be useful to consider the discrimination over the whole frequency range. As pointed out in Section 1, when the unwanted frequency ω_2 is remote from ω_1, then it receives no positive feedback although ω_1 has the full amount. There is thus a frequency discrimination against ω_2, coupled with a smaller amount of non-linear amplitude discrimination if the amplitude $\hat{E}_{L2}$ is less than $\hat{E}_{L1}$. As ω_2 approaches more nearly to ω_1 it receives an increasing amount of feedback, and the discrimination against it is reduced. Finally, when ω_2 is very close to ω_1, it has only amplitude discrimination.

A numerical example will make this clear. Taking $a_1/a_3 = -5.75$, $\hat{E}_{g0} = 1.0$, and $Q = 15$, we have, assuming $\hat{E}_{L2}^2 \ll \hat{E}_{L1}^2$:

Case (1). If $\hat{E}_{L1} = 0.1$, then from Fig. 4, $\hat{E}_{g1} = 1.24$ The gain which ω_1 receives due to feedback is therefore 21.9 db. There is also, from Fig. 2, 2.7 db discrimination against ω_2 due to non-linearity. The total discrimination when $\omega_2 \ll \omega_1$ or $\omega_2 \gg \omega_1$ is therefore 24.6 db. When $\omega_2 \approx \omega_1$ the discrimination is 14.2 db, from Fig. 6. In the intermediate range the gain provided by the feedback effect to the frequency ω_2 is given [1,3] by

$$\left|\frac{\hat{E}_{g2}}{\hat{E}_{L2}}\right| = \sqrt{\left[\frac{1 + Q^2(1 - x^2)^2}{(1 - x)^2 + Q^2(1 - x^2)^2}\right]} \quad (12)$$

where $x = \omega_2/\omega_1$. This ignores the fact that the feedback loop is non-linear, since the consideration of this factor makes the equations practically insoluble.[1]

On plotting these relations, it is easy to join the three ranges smoothly together, giving an approximate overall response curve as shown in Fig. 7, which must necessarily give a close approach to the curve in spite of the difficulty just mentioned.

Case (2). If $\hat{E}_{L1} = 0.03$, then $\hat{E}_{g1} = 1.09$. Thus when $\omega_2 \ll \omega_1$ or $\omega_2 \gg \omega_1$ the discrimination is $31.2 + 2.0$ db. When $\omega_2 \approx \omega_1$ it is 19.6 db (from Fig. 6). The overall response is also shown in Fig. 7.

It will be seen, therefore, that the response is throughout dependent on the actual voltages in the circuit, but if the unwanted signal is of considerably lower amplitude than the synchronizing signal in the input, then the discrimination is dependent only on the voltages of the synchronizing frequency. The important conclusion is that there is everywhere some discrimination, a property not possessed by any linear frequency-selective circuit.

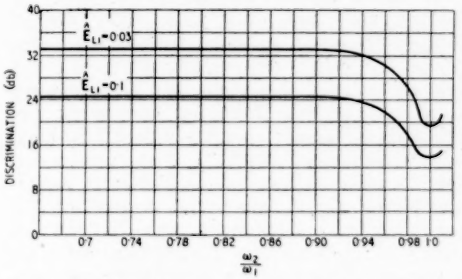

Fig. 7. *Discrimination over the whole frequency range* ($\hat{E}_{g0} = 1.0$, $a/a_3 = -5.75$, $Q = 15$).

2.6 *Output from the Tuned Circuit*

We have so far considered the output as being taken from the anode of the valve, with the feedback resistance large compared with the anode resistance, as shown in Fig. 1. If the output were taken across the tuned circuit, as is quite common practice, the performance would be slightly modified, in that the frequency response of the tuned circuit itself must be added to the discrimination shown in Fig. 7.

2.7 *Practical Design of the Circuit*

The analysis has shown how the circuit behaves; in practice, we have to consider how best to use this behaviour for meeting specified requirements. To reduce the depth of modulation to a specified extent, it will be necessary to obtain from a family of curves such as Fig. 6 suitable values of $\hat{E}_{g0}$ and $\hat{E}_{L1}$ (assuming, of course, that some particular valve is already decided upon). The value of $\hat{E}_{g0}$ should not be unnecessarily high, as this might lead to difficulty with the third-order products, as discussed in Section 2.4. The value of $\hat{E}_{L1}$, in relation to $\hat{E}_{g0}$ and the Q-factor of the tuned circuit, determines over what frequency range the oscillator will remain synchronized; if the input is unmodulated, the relationship for small values of $\hat{E}_L$ is as follows[3]:—

$$\hat{E}_L = 2Q \cdot \hat{E}_{gp} (1 - x_p) \quad .. \quad .. \quad (13)$$

where $\hat{E}_{gp}$ is the grid voltage at the pull-out point (this is somewhat less than $\hat{E}_{g0}$, by an amount not exceeding about 30%), and x_p is the ratio of the pull-out frequency to the frequency of free oscillation.

If the input is modulated, the value of $\hat{E}_L$ used must be that at the troughs of the modulated envelope.

The synchronous range must cover the normal range of variation of frequency of the oscillator, and thus the value of $\hat{E}_{L1}$ chosen must satisfy this condition also. The Q-value of the tuned circuit does not influence this requirement, because the stability of the oscillator is approximately proportional to Q, but it does affect the discrimination against unwanted frequencies at some parts of the frequency range, as discussed in Section 2.5.

3. Byard and Eccles' Experimental Results

In view of the previous publication of experimental observations on this subject by Byard and Eccles,[2] it seems only just to consider whether their results are consistent with the theory developed here. The experimental results will be discussed in terms of the diagram numbers of the published paper. It should be observed that Byard and Eccles use a somewhat different circuit from that considered in the present paper, and their "output" is taken from the grid circuit. This means that only the feedback effect of Section 2.2 is involved.

Fig. 2.—An increase in the depth of modulation is indicated as the oscillator is tuned to a frequency less close to that of the input tone. This is quite reasonable, since the amount of feedback is thereby reduced, thus decreasing the reduction in depth of modulation.

Fig. 3.—The linear relation between output and input depths of modulation is to be expected from equation (3), provided the depth of modulation is not excessive. It is surprising that the linear relationship appears to hold so closely even up to 100% modulated input. The increase in modulation of the output as the input level is increased is to

be expected from a consideration of Fig. 5 of the present paper.

Figs. 4 and 5.—It is only to be expected that the synchronous frequency range will be determined by the troughs of the modulated input signals, and the linear relation is in agreement with the present author's general theory of synchronization[3].

Fig. 6.—The increase in modulation of the output as the input level is increased is to be expected (as mentioned above under Fig. 3); the fact that this figure shows a linear relationship suggests that the range of input voltage is restricted. Since the actual range of input voltage shown is from zero to a value slightly in excess of the free oscillation voltage, it must be concluded that the valve used had a different (i.e., lower) ratio a_1/a_3 from that considered in the examples in the present paper.

Fig. 7.—The decrease of the depth of modulation as the feedback is increased with constant input voltage correctly corresponds to the theory, as can be seen from Fig. 5 of the present paper.

It can thus be concluded that Byard and Eccles' experimental results are consistent with the theory here presented.

4. The Non-linear Response of Actual Valves

It has been assumed above that the valve response can be adequately represented by a cubic relationship between output and input voltage. In practice, of course, the relationship is much more complicated, and it is difficult to match the response even by a fifth-power expression. But the general nature of the behaviour of the circuit is quite accurately demonstrated by the use of the cubic, and the author has, in fact, found little serious discrepancy between theory and experiment due to using the cubic law as far as the feedback effect is concerned. The direct effect (see Section 2.1) gives poorer agreement. The best plan is to determine a suitable cubic law covering the usually small range of voltage values used in practice. For instance, the ratio of $a_1/a_3 = -5.75$, which has been used in the numerical examples above, represents approximately a valve type SP41 (Mazda) used with an H.T. voltage of 150 volts and a grid voltage (peak a.c.) range from 0 to about 1.5 volts.

5. Conclusions

Amplitude discrimination in non-linear circuits, whereby a signal of high level can reduce the output of a signal of lower level, can be made a useful and readily utilizable effect by the application of positive feedback sufficient to produce self-oscillation in the absence of an input signal. The system then becomes a synchronized or "locked-in" oscillator, which is shown to provide discrimination against signals mixed with the locking tone not only when they are of frequency lying outside the synchronous range (an effect due chiefly to frequency-selectivity), but also when they lie extremely close to the synchronizing frequency (an effect due entirely to non-linearity coupled with positive feedback). Thus, if a modulated tone is injected into an oscillator as a synchronizing signal, then the output of the oscillator will contain a greatly reduced depth of modulation. This system provides a means of obtaining a relatively pure tone from a signal which may have other frequencies mixed with it which cannot be separated by ordinary frequency-selective circuits. The only restriction needed is that the unwanted signal component in the input must be smaller than the wanted component. The effect is not dependent on the unwanted frequencies being sidebands of an amplitude-modulated wave.

Previously published experimental work by Byard and Eccles is shown to be consistent with the theory developed here.

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[3] D. G. Tucker, "Forced Oscillations in Oscillator Circuits, and the Synchronization of Oscillators," *Journ. Instn elect. Engrs*, 1945, Vol. 92, Part III, p. 226.

CORRESPONDENCE

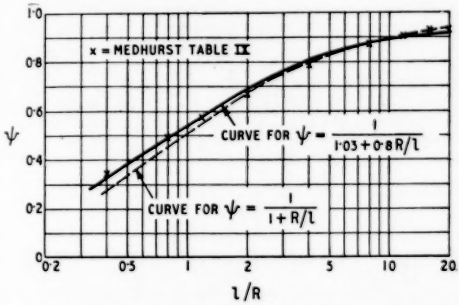
Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Q of Solenoid Coils

To the Editor, "Wireless Engineer"

SIR,—From Mr. Medhurst's paper in *Wireless Engineer* for March, one concludes that the results of his careful measurements on solenoid coil Q should be taken as superseding Butterworth's formulæ which have for so long been taken by engineers as a practical basis for coil design.

Mr. Medhurst's results for the Q of coils wound with the optimum gauge of wire are contained in his formula $Q = 0.15R\psi\sqrt{f}$ and his curve for ψ shown in Fig. 13. It is interesting to note that the latter curve agrees to an accuracy of a few per cent with the simple empirical equation $\psi = 1/(1.03 + 0.8R/l)$ as shown in the graph. Thus we arrive



at the following simple and practical formula for the Q of a properly designed solenoid, using plain copper wire, at high radio frequencies:—

$$Q = \frac{\sqrt{f}}{(6.9/R + 5.4/l)}$$

where R and l are the radius and length of the coil in centimetres. In a majority of practical cases we can use the even simpler formula

$$Q = 0.15\sqrt{f}/(1/R + 1/l)$$

which follows the data to a few per cent, provided $l > R$.

The range of conditions under which this formula applies is the same as that to which Mr. Medhurst's data refer: in particular—

(*a*) The ratio of wire diameter to wire spacing must approximate to the optimum (i.e., this ratio should lie between 0.5 and 0.7 for short coils ($l < 2R$) or 0.6 to 0.8 for l order of $4R$, and 0.75 to 0.9 for very long coils).

(*b*) The formulæ apply strictly only for very high frequencies, but (from Sect. 10) the accuracy will be better than ± 10 per cent provided $z > 7$; i.e., provided frequency in Mc/s exceeds 0.5 divided by (wire diameter in mm)².

Solenoids are chiefly used in current practice on frequencies above about 3 Mc/s, and here Litz wire is of little or no advantage; the following table, giving the thinnest gauge for which the formula applies to ± 10 per cent, shows that most practical solenoids will be covered.

freq.	1 Mc/s	4 Mc/s	16 Mc/s
wire ..	22 S.W.G.	28 S.W.G.	37 S.W.G.

(*c*) The formulæ do not hold for coils of very few turns (or extremely short coils). Mr. Medhurst gives us little guidance as to how far Butterworth's correction factor for coils of few turns can be relied on. Further experimental work seems indicated in view of the practical importance of coils of very few turns on v.h.f. and u.h.f.

(*d*) Dielectric loss is not, of course, allowed for. This is unlikely to be material except where the coil has a rather poor dielectric (bakelite or worse) and is used in a circuit having a low parallel tuning capacitance.

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Absolute Rotation and a Rotating Magnet

To the Editor, "Wireless Engineer"

Sir,—There are two aspects of the old rotating magnet problem: one is the absolute property of rotation, and the other is the electromagnetic reaction on the magnet itself. Various suggestions have been put forward by different authors to explain absolute rotation and from them to interpret the electromagnetic reaction upon the magnet.[1]

The problem is, however, as fundamental as it is hoary. We should especially notice that the questions of absolute acceleration and absolute rotation, which were sanctioned in the special relativity and which signify a return to the Newtonian Hypothesis of absolute space and absolute time, served, in fact, as the genesis of Einstein's theory of general relativity. Einstein's "principle of equivalence" asserts that, for the description of physical processes, we may take either the view-point of an observer A, at rest with an inertial coordinate system or that of another observer B, on a constantly accelerated coordinate system who measures the physical processes by means of the coordinates of observer A and, in addition, assumes the presence of a homogeneous gravitational field.[2] This principle, properly applied here for any particular instant, gives us a simple way of explaining so-called "absolute rotation". The electromagnetic reaction upon a rotating magnet and the resultant electric field distribution in it can then be obtained from Maxwell's equations modified for the relativity effect.

1. *Absolute Rotation*

Consider an inertial system Σ, in which the position-vector of any point is given by

$$\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z \quad .. \quad .. \quad .. \quad .. \quad (1)$$

and another coordinate system Σ', with its $\mathbf{k}'$-axis and origin coinciding with the $\mathbf{k}$-axis and origin of the first system, and rotating about the $\mathbf{k}$-axis with constant angular velocity ω. The position-

vector for the same point referred to the latter system is then

$$\mathbf{r} = \mathbf{i}'x + \mathbf{j}'y + \mathbf{k}'z \quad .. \quad .. \quad .. \quad (2)$$

Denoting differentiation with respect to the rotating system by a prime ($'$), then the acceleration of the position-vector for both systems is given by the following relation[3]:

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d'^2\mathbf{r}}{dt^2} + 2\left[\boldsymbol{\omega} \times \frac{d'\mathbf{r}}{dt}\right] + [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] \quad (3)$$

The second term on the right-hand side is the Coriolis acceleration, which is present only when the particle moves in the rotating system. Let m be the mass of the particle considered, then Newton's second law of motion gives

$$\mathbf{F} = m\frac{d^2\mathbf{r}}{dt^2} = m\frac{d'^2\mathbf{r}}{dt^2} + 2m\left[\boldsymbol{\omega} \times \frac{d'\mathbf{r}}{dt}\right] + m[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] \quad .. \quad .. \quad .. \quad (4)$$

The force referred to the moving system is

$$\mathbf{F}' = m\frac{d'^2\mathbf{r}}{dt^2} = \mathbf{F} - 2m\left[\boldsymbol{\omega} \times \frac{d'\mathbf{r}}{dt}\right] - m[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] \quad .. \quad .. \quad .. \quad (5)$$

This is zero since $\mathbf{r}$ is held constant both in magnitude and in direction with respect to the rotating coordinate system. The Coriolis force also disappears for the same reason. Thus

$$\mathbf{F} = m[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] = m\omega^2 r\mathbf{n} = m\mathbf{a} \quad .. \quad (6)$$

represents the force which is necessary to hold the particle in rotation and which is constantly changing in direction, and $\mathbf{a}$ gives the centripetal acceleration as shown in Fig. 1.

Now an observer in the rotating coordinate system Σ' with a centripetal acceleration, but unconscious of the force $\mathbf{F}$, will report the existence of a gravitational field corresponding to the familiar centrifugal force

$$\mathbf{F}' = -m[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] = -m\omega^2 r\mathbf{n} \quad .. \quad (7)$$

This is nothing but Einstein's principle of equivalence applied to a rotating system for the observer B at any instant. The observer B in the rotating system Σ' will then use the coordinate language of his comrade observer A in the rest inertial system Σ for the description of the physical processes in addition to the presence of this gravitational field $\mathbf{F}'$, and ascribe a constant tangential velocity

$$|\mathbf{v}| = |\boldsymbol{\omega} \times \mathbf{r}| = |\omega r\mathbf{t}_1| = \omega r \quad .. \quad .. \quad (8)$$

to the particle at any instant. In fact, the observer B will attribute his gravitational field $\mathbf{F}'$ to the cause of rotation with constant angular velocity ω if he uses the same physical and mathematical language as we use. This explains therefore the so-called "absolute property of rotation."

2. Electromagnetic Reaction on a Rotating Magnet

Let the observer B mounted on the rotating magnet (Σ'-system) be armed with a unit positive electric charge e. Since he is moving with a constant linear velocity ($v = \omega r$) in a stationary magnetic field produced by the magnet, he will experience an "electric force" at any instant given by [4].

$$\mathbf{F}'_e = e\mathbf{E}' = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}[\mathbf{v} \times \mathbf{B}] \quad .. \quad (9)$$

in a direction perpendicular to the magnetic flux density $\mathbf{B}$ and to his direction of motion at that instant. As shown in Fig. 1, at the instant considered, the rotating co-ordinate-axes x, y, z, coincide with those of the inertial system x, y, z and the rotating system is turning about the $\mathbf{k}$- or $\mathbf{k}'$-axis with a constant angular velocity ω. Let the observer armed with the unit positive

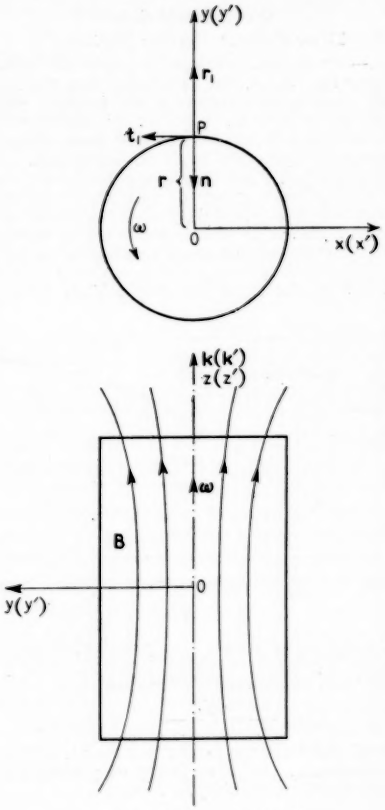

Fig. 1

charge be at point P on the y-axis. Then his velocity will only have a component along the negative x-axis, that is

$$\mathbf{V} = -v\mathbf{i} + o\mathbf{j} + o\mathbf{k} \quad .. \quad .. \quad .. \quad (10)$$

The magnetic field $\mathbf{B}$, on the other hand, can have no component along the x-axis from circular symmetry, that is

$$\mathbf{B} = o\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} \quad .. \quad .. \quad .. \quad (11)$$

Substituting relations (10) and (11) into (9), we obtain the components for the electric field experienced by the observer B: viz.,

$$E'_x = o,\ E'_y = \frac{vB_z}{\sqrt{1 - \left(\frac{v}{c}\right)^2}},\ E'_z = \frac{-vB_y}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad .. \quad .. \quad .. \quad (12)$$

Since the magnetic field is practically parallel to the z-axis, E'_z is negligible compared with E'_y. Thus the movement of charges inside the magnet is for the most part due to E'_y, while the small E_z'

component is effective only at both end-edges. The charge distribution thus arising can easily be calculated if we make the reasonable assumptions, to wit

$v/c \ll 1$, $B_y = 0$ and $B_z =$ constant (inside the magnet).

Relations (12) then simplify to

$$E'_x = 0,\ E'_y = vB_z = \omega r B_z,\ E'_z = 0 \quad .. \quad (12a)$$

From the evident circular symmetry, the field quantities are best expressed in terms of cylindrical coordinates; that is

$$\mathbf{B} = B_z\mathbf{k};\ \mathbf{E}' = E'_r\,\mathbf{r}_1 = vB_z\mathbf{r}_1 = \omega r B_z\,\mathbf{r}_1 \quad \ldots \quad .. \quad .. \quad (13)$$

The charge density produced inside the rotating magnet is then given by the divergence of the dielectric displacement; viz,

$$\rho = \text{div. } \mathbf{D}' = \text{div. } (\epsilon\mathbf{E}') = \epsilon\left[\frac{1}{r}\frac{d}{dr}(rE'_r)\right]$$

$$= \frac{e}{r}\frac{d}{dr}(\omega^2 r^2 B_z) = 2\epsilon\omega B_z \quad .. \quad .. \quad (14)$$

We thus have constant space charge density of positive charges in the magnet and a line-charge of negative charges along the axis of rotation. The existence of the positive space charge in a rotating magnet is susceptible to experimental verification by its action upon a test charge nearby. At the same time the positive space charge, rotating with the magnet, will produce a convection current strengthening the magnetic field of the magnet.

3. *Forces acting upon a Rotating Test-Charge outside a rotating Magnet*

From previous considerations it is evident that, if a test-charge $+q$ outside the magnet is rotating together with the magnet with the same angular velocity ω, it will be subjected to three electromagnetic forces, namely,

(*a*) An electrostatic Coulomb force upon the test-charge $+q$ caused by the space-charge distribution inside the magnet which itself arises from the magnet's rotation, the space-charge at any point being given by Equ. (14). This force is directed radially outward and is denoted by $\mathbf{F}_{1q}$ in Fig. 2.

(*b*) Part of the magnetic force upon the test-charge convection-current $+q\mathbf{v}_q$ due to the magnetic field

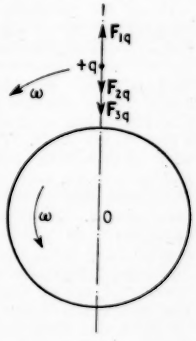

Fig. 2

produced by the rotating space-charge in the rotating magnet. This is directed radially inward and is shown in Fig. 2 as $\mathbf{F}_{2q}$.

(*c*) Part of the magnetic force upon the test-charge convection-current $q\mathbf{v}_q$ due to the original stationary magnetic field produced by the magnet at rest. This force is denoted by $\mathbf{F}_{3q}$ in Fig. 2 and is also directed radially inward.

The above simple considerations tend to throw some light upon the complexion of an electron moving along its orbit about the nucleus in an atom. In addition to the Coulomb force, each electron is definitely subjected to a sort of magnetic reaction due to the magnetic field produced inside an atom by other moving electrons and also the nuclear motion, especially for heavy atoms.

National Central University, Nanking. CHANG-PEN HSÜ (Professor of Electrical Engineering.)

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Permeability of Dust Cores

To the Editor, "Wireless Engineer"

SIR,—I have read with interest the correspondence and Editorials on this subject and I believe that there is a difference of principle between toroidal cores of high permeability and toroidal cores made from the material used for high-frequency cores. The latter contain much more insulation and binder, and are pressed with less than 10% of the tonnage required for the high-permeability cores; it can be said in general that the particles in such cores will not be deformed.

Having made microscopic examinations of high-permeability toroidal cores on many occasions I have, as yet, never seen the aligned particles which Mr. Bardell claims, nor have I been able to observe his slab formation with the dimensional ratio of 10 : 2 : 1. The idea itself is not new and is contained in the British Patent Specification No. 342,999 of 1929 which claims the magnetic adjustment of oblong particles inside the mould before pressing. I assume that Polydoroff's application of the brush-production technique to cores as described in the British Patent Specification No. 557,838 may also be applicable. Originally suggested for long cores, there may be difficulties with toroids.

It has, on the other hand, occurred to me that there may be another cause for the observed increase of permeability. All formulae are based on the assumption that the particles have a reasonably regular shape and that the airgaps are evenly distributed. Every microscopic examination has so far shown two facts:

1. The very soft (annealed) material will "flow" under pressure and the boundary airgap round the now very irregular shaped particle will be about half a micron;

2. Relatively large airpockets can be observed on every section of the toroidal core under examination.

The second point probably needs some explanation. It is due to the effect known as "bridging" in powder metallurgy. The insulated powder has to be shaken into the mould, a vessel with limited space. In this process the powder is liable to form arches. The space under the arches is void of powder and the arches themselves resist pressure, retaining an airpocket in the core. The bridging depends partly on the particle size and shape, but also on the insulation and atmospheric conditions. As they are distributed unevenly, some airpockets will lie across some lines of force while others will not be affected. The effect will be that the particles will be separated by a smaller airgap than that calculated from the percentage of magnetizable material present in the core and its weight. These smaller airgaps should not only

have an effect on the permeability but also on the hysteresis coefficient. This assumption seems to agree with the thread-like alignment suggested elsewhere.

There is another point : Instead of going through the airpocket the affected lines will go round it, and thus increase the induction in the particles adjacent to such pockets. An increased induction, however, (we are far from the saturation point) will create locally all those phenomena normally associated with an increased field. I wonder therefore whether this and the aforementioned consideration will not explain both the increased permeability and part of the residual loss as far as this is due to inaccuracies in our methods for loss separation.

Dr. Bardell's judgment on Legg & Given's figures stands on very weak feet. His own figures can be considered as arbitrary as he does not mention a basis on which his figures and the other ones can be compared. If the intrinsic permeability is measured with direct current it is certainly necessary to link the permeability to the current. Speed & Elmen in their 1921 paper on " Magnetic Properties of Compressed Powdered Iron " have published interesting tables on measurements made on toroids from uninsulated powder, pressed under different tonnage. They show how the permeability of such cores is affected by the magnetic test conditions (due to the considerable wider hysteresis loop) and make it unnecessary to repeat figures.

It is, however, doubtful whether either of the authors has actually measured the true intrinsic permeability in this way. All powders are annealed in a reducing atmosphere. Even if they were supplied in containers filled with inert gas (and they are not), the powder with its relatively large surface area is exposed to air during the weighing, filling of the die, levelling and closing. This is long enough, and the material is sufficiently stirred up to be oxidized. It will depend on the oxide formed and its magnetic and mechanical properties how near the measured figures are to the permeability of the powder particle. The oxide may have magnetic properties or not, it may be brittle or it may be tough enough to resist a certain pressure. It may be pierced and it may only be dented, thus altering the conductivity as compared with the particles which form direct contact. That these considerations are important is shown by the fact that oxidization by steam and water can result in reasonable insulation (Speed & Elmen).

Ferrocart A which was mentioned in Professor Howe's Editorial has very quickly been replaced by moulded grades and it is doubtful whether anything more than samples have reached this country. The replacement took place probably in 1933.

Manchester. E. R. FRIEDLAENDER.

The Equivalent Diode

To the Editor, " Wireless Engineer "

SIR,—The article by G. B. Walker on the theory of the equivalent diode in your January issue, and particularly his remark on the difficulty of obtaining experimental verification, has led me to make a comparison of the expression which he derives, that obtained by Fremlin,[1] and one due to Tellegen,[2] with the intention of finding what conditions are necessary for a critical experiment.

The expressions may all be written in the form

$$I = 2.34 \times 10^{-6} \frac{(V_a + \mu V_g)^{\frac{3}{2}}}{g^2 (1 + \mu)^{\frac{3}{2}} B}$$

where, with $s = a - g$,

$$B_W = \left(1 + \frac{s}{g\,(1 + \mu)}\right)^2$$

$$B_F = \left(1 + \frac{\left(1 + \frac{s}{g}\right)^{\frac{3}{2}} - 1}{1 + \mu}\right)^{\frac{2}{3}}$$

$$B_T = \left(1 + \frac{4}{3}\frac{s}{g\,(1 + \mu)}\right)^{\frac{3}{2}}$$

Since $s/g\mu$ may be written p log (coth $\pi\, d/p)/2\pi g$ where p is the pitch of the grid and d the diameter of its wire, and since p/g must be less than unity to avoid " Inselbildung " $s/g\,(1 + \mu)$ depends mainly on d/p, is larger for widely-spaced fine wires, and will be less than unity for any reasonable structure. Hence B_T may be expanded by the binomial theorem

$$B_T = 1 + \frac{2s}{g\,(1 + \mu)} + \frac{2}{3}\left(\frac{s}{g\,(1 + \mu)}\right)^2 - \frac{4}{27}\left(\frac{s}{g\,(1 + \mu)}\right)^3 + \frac{2}{27}\left(\frac{s}{g\,(1 + \mu)}\right) - \ldots$$

$$B_W = 1 + \frac{2s}{g\,(1 + \mu)} + \left(\frac{s}{g\,(1 + \mu)}\right)^2$$

These expansions are remarkably similar, and differ by 10 per cent only when $s/g(1 + \mu)$ is nearly unity, so that it would be very difficult to choose between the formulæ of Walker and Tellegen experimentally.

It is not possible to expand B_F in terms of integral powers of s/g if s/g is greater than unity, but by taking particular cases it may be seen that the difference between B_F and B_W is greater than that between B_T and B_W, and that it could be made large enough to permit of experimental discrimination with reasonably fine grid wires and a reasonable pitch. The structure used by Fremlin, with $d = 0.0089$ cm, and $p = 0.175$ cm, has $B_F/B_W = 1.12$, and this ratio could be increased to 1.2 by using wire of half the diameter and increasing s. Unfortunately Fremlin's very careful experiment is vitiated, because he neglected the factor[3] $[1 + 0.024T^{\frac{1}{2}}\,(V - V_m)^{-\frac{1}{2}}]$, where T is the absolute temperature of the cathode, V is the anode voltage of the equivalent diode, and V_m is the voltage of the potential minimum, which corrects the three-halves law for the existence of initial electron velocities, and with the voltages which he used this had an average value of about 1.25. (The agreement obtained between theory and experiment was probably due to a compensating error introduced by the incorrect method of allowing for contact potential.) With an apparatus essentially similar to that used by Fremlin, but using finer grid wire and with an anode voltage of about 1,000, it would be possible to reduce the initial velocity corrections below 10 per cent for a range of grid volts and thus determine which formula best fits the facts. Special methods of cooling the anode would be needed. The fact that such a highly unusual triode would be necessary

is an indication that the matter is of academic interest only.

A point of theoretical interest arises from the above work. Walker tests the validity of the formulæ he discusses in the extreme cases when μ tends to infinity or zero. Tellegen's formula fails in the latter case, though it does not tend to zero. A similar but slightly more complicated test is to consider the anode removed to a great distance. Since μ is proportional to s, $s/g(1+\mu)$ remains finite, but

$$\left(1+\frac{s}{g}\right)^{\frac{3}{2}}/(1+\mu)$$

tends to infinity.

Thus the current predicted by Walker and Tellegen tends as we would expect to something a little less than

$$2.34 \times 10^{-6} V_g^{\frac{3}{2}} g^2,$$

whereas that given by Fremlin tends to zero.

It may be concluded that an experimental discrimination between the formulæ of Walker and Tellegen is not possible, that an experimental choice between these two and Fremlin's though possible, is not worth the trouble, but that Walker's approach is to be preferred, because it is theoretically sounder, because it is not dependent on the three-halves law and can thus be used in calculations where initial velocities are taken into account, and because it has the important virtue of simplicity.

J. Eastabrook,
Standard Telephones and Cables, Ltd.
Ilminster, Somerset.

[1] *Philosophical Magazine*, 1939, Vol. 27, p. 709. *Electrical Communication*, July 1939.
[2] *Physica*, 1925, Vol. 5, p. 301.
[3] Langmuir I, *Physical Review*, 1923, Vol. 21, p. 419.

I.E.E. RADIO SECTION

The annual dinner of the Radio Section of the Institution of Electrical Engineers was held at the Café Royal, Regent Street, London, on 30th April.

Prof. Willis Jackson, Chairman of the Section, announced that Mr. H. J. Nunn was relinquishing his position as secretary of the Radio Section and that it would be taken over by Mr. K. W. T. Brown. A presentation was made to Mr. Nunn in appreciation of his services and it was pointed out that he had not only been secretary of the Section for fifteen years but had been associated with the Institution for forty.

INSTITUTION OF ELECTRONICS

The second exhibition of electronic apparatus will be held at the Great Hall, College of Technology, Manchester, on 22nd July at 2.30–9 p.m. and on 23rd July at 10 a.m.–9 p.m. Admission is by ticket and non-members should apply to A. Coates, 16, Didsbury Park, Manchester, 20.

NEW E.R.A. SECTIONAL COMMITTEE

A new sectional committee of the Electrical Research Association has been set up with as terms of reference the studying and developing of the electrical equipment of automotive systems with particular reference to electrical ignition systems, radio interference suppression devices and electrical accessories. Under the chairmanship of Dr. E. A. Watson, the committee comprises representatives of the Ministry of Supply, the Admiralty, the Motor Industry Research Associations, the British Internal Combustion Engine Research Association, the Department of Scientific and Industrial Research and most of the leading manufacturers of automobile electrical components.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

581 125.—Radiating intense waves or beams of audible frequency from a plate-resonator carrying standing-waves created by a piezo-electric or magnetostrictive drive.

Marconi's W.T. Co. Ltd. and H. J. Round. Application date 7th November, 1941.

581 510.—Microphone amplifier, with a readily-adjustable reed, suitable for a deaf-aid set.

Fortiphone Ltd. and L. M. Seath. Application date 10th August, 1944.

AERIALS AND AERIAL SYSTEMS

580 569.—Aircraft aerial comprising a sphere or hemisphere of dielectric material with metallic dipoles or slot radiators.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 21st April, 1944.

580 750.—Directive aerial system comprising two elements, each with two diverging conductors, driven in phase-quadrature.

Marconi's W.T. Co. Ltd. (assignees of C. W. Hansell). Convention date (U.S.A.) 27th January, 1943.

581 570.—Eccentric driving-gear for rotating a parabolic reflector so that the axis of the radiated beam generates a small-angled cone in space.

E. C. Cork and M. Bowman-Manifold. Application date 15th February, 1943.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

580 481.—Rotary-drum keying arrangement for the aerials of a blind-approach radio-navigational system using ultra-short waves.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 11th August, 1941.

580 482.—Radiolocation equipment with means for the automatic correction of marginal errors made when following a moving target.

The British Thomson-Houston Co. Ltd. and T. H. Mackenzie. Application date 3rd April, 1942.

580 640.—Blind-landing system in which a tangential glide-path is formed by radiators located fore and aft of the landing point.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 5th March, 1943.

580 687.—Scanning system for televising the indications received by radiolocation equipment to a remote point.

B. J. Edwards and Pye Ltd. Application date 13th August, 1943.

580 975.—Radiolocation set in which the transmitting and receiving aerials scan the field-of-view in quadrature so as to determine two angular co-ordinates of the target.

Sperry Gyroscope Co. Ltd. and R. H. Nisbet. Application date 8th November, 1940.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

580 402.—Reducing "shot noise" in an electronic tube utilizing velocity-modulation for superheterodyne reception.

C. S. Bull. Application date 2nd September, 1942.

580 815.—Piezo-electric discriminator circuit for detecting phase-modulated carrier-signals.

Marconi's W.T. Co. Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.), 16th February, 1943.

580 905.—Time-controlled relay for switching a wireless receiver on and off at predetermined times, and a local tone-generator which allows the device to be used as an alarm-clock.

J. Carter. Application date 3rd October, 1944.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

580 413.—Television receiver of the kind in which a scanning-beam varies the opacity of a fluid layer, thereby controlling the passage of light through it from an external lamp.

Gesell. zur Forderung & Technischen Hochschule. Convention date (Switzerland) 5th June, 1943.

580 860.—Cathode-ray tube designed to secure a more sensitive control of the electron beam for line-scanning than for frame-scanning, say in television.

L. F. Broadway. Application date 9th March, 1944.

580 887.—Television system in which pulsed voltages are applied to the c.r. signal-developing tube, to increase its operating efficiency.

H. G. Lubszynski. Application date 16th March, 1944.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

580 738.—Generating a frequency-modulated signal from a time-modulated train of pulses, the carrier-wave being developed from an harmonic of the train.

Standard Telephones and Cables Ltd. and C. T. Scully. Application date 23rd April, 1943.

580 769.—Generating pulse-trains with time-phase and time-duration modulation, wherein the unmodulated interval is short compared with the repetition period of the pulses.

Standard Telephones and Cables Ltd., P.K. Chatterjea and C. T. Scully. Application date 2nd May, 1944.

580 801.—Generating pulses at a frequency of say 15 kc/s, each pulse occupying only 0.3 per cent of the repetition time.

Standard Telephones and Cables Ltd. and W. A. Beatty. Application date 27th February, 1940.

580 843.—Secret signalling-system based upon the rise of an amplitude-modulated train of pulses in combination with a masking signal.

Standard Telephones and Cables Ltd., P. K. Chatterjea and L. W. Houghton. Application date 18th December, 1941.

580 930.—Multiple wired-wireless system in which each transmitter is tuned, to serve as a wave-trap to shut-out intruding signals.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 27th July, 1943.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

580 164.—Electrode arrangement and deflecting means for generating very short waves in a valve of the beam type.

Marconi's W.T. Co., Ltd., and C. S. Franklin. Application date 13th March, 1941.

580 730.—Cathode-ray tube in which the electron beam is arranged to produce an illuminated area with a dark centre-spot to increase the accuracy of definition.

The General Electric Co. Ltd. and L. C. Jesty. Application date 14th September, 1942.

580 964.—Hollow resonator, as used for velocity-modulation, provided with a tuning device in the form of a rotary vane.

N. C. Barford. Application date 7th September, 1943.

SUBSIDIARY APPARATUS AND MATERIALS

579 940.—Rotary transformer and balanced-bridge arrangement for stabilizing the frequency of an electric power-supply system.

F. R. Milsom, Furzehill Laboratories, Ltd. and S. Smith & Sons (Motor Accessories), Ltd.). Application date 14th March, 1944.

580 044.—Valve relay circuit operated by the movement of a magnetic core; e.g., for controlling liquid levels or pressures.

Igranic Electric Co., Ltd., J. M. Bedford, J. A. Field and S. P. Maynard. Application date 24th January, 1944.

580 377.—Construction of a flexible waveguide of rectangular cross-section.

C. E. Fenwick and C. S. Wright. Application date 28th March, 1944.

580 527.—Integrating circuit in which negative reaction is utilized to increase the normal time-constant: applicable for generating saw-tooth voltages.

A. D. Blumlein. Application date 5th June, 1942.

580 824.—Crystal-controlled oscillation-generator in which an auxiliary inductance serves to offset the normal falling-off in amplitude of the higher harmonics.

Marconi's W.T. Co. Ltd. and G. L. Grisdale. Application date 8th May, 1944.